
Narrowband CMOS RF Low-Noise Amplifiers

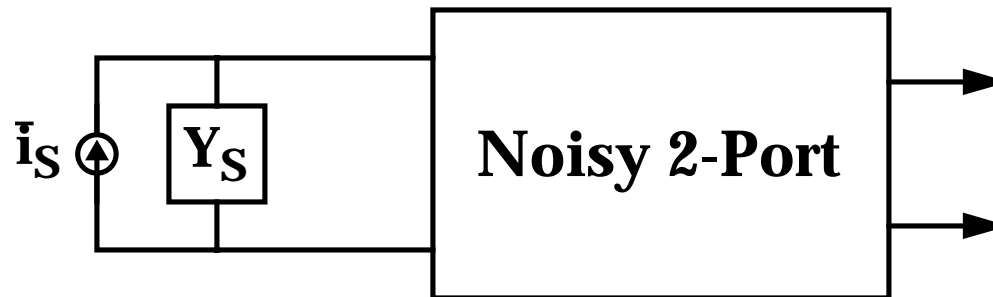
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Outline

- ❑ **A brief review of classic two-port noise optimization**
 - ❑ **Conditions for minimum noise figure**
 - ❑ **The fundamental importance of correlations**
- ❑ **MOSFET noise models in the short-channel regime**
 - ❑ **Equivalent two-port noise generators**
 - ❑ **Second-order noise sources**
- ❑ **Power constrained noise optimization**
- ❑ **Experimental results on devices and circuits**
- ❑ **Summary and conclusions**

Classic Two-Port Noise Optimization

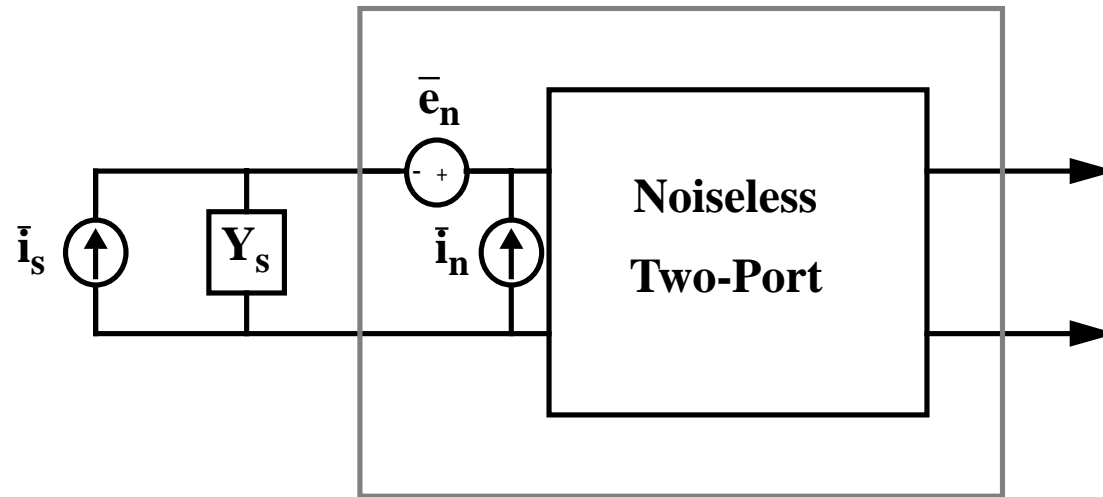
- ❑ Consider noise in an arbitrary (but linear) system:



- ❑ Thermal noise of source represented by \bar{i}_S
- ❑ Source admittance is Y_S

Classic Two-Port Noise Optimization

- ❑ The noisy two-port may be modeled as follows:



- ❑ In general, the external noise sources will be partially *correlated*
 - ❑ Correlations arise because an internal noise source may contribute to both \bar{i}_n and \bar{e}_n in general
 - ❑ Correlations have strong implications for noise performance

Classic Two-Port Noise Optimization

- ❑ Noise factor, F , is defined as the ratio of the total output noise power divided by that part of the output noise power due to the input source, when source is at 290K
- ❑ Therefore:

$$F = \frac{\overline{i_s^2} + \overline{|i_n + Y_s e_n|^2}}{\overline{i_s^2}}$$

- ❑ Let noise current i_n be expressed as sum of two terms
 - ❑ First term, i_u , is fully uncorrelated with noise voltage e_n .
Other term, i_c , is fully correlated with e_n .

Classic Two-Port Noise Optimization

- Since i_c is correlated with e_n , we may write one as proportional to the other:

$$i_c = Y_c e_n$$

- Note that Y_c has the dimensions of an admittance
 - Correlation admittance is a mathematical construct, and is not what one measures with an impedance meter
- Re-write F as

$$F = 1 + \frac{\overline{|i_n + Y_S e_n|^2}}{\overline{i_S^2}} = 1 + \frac{\overline{i_u^2} + \overline{|Y_c + Y_S|^2 e_n^2}}{\overline{i_S^2}}$$

Classic Two-Port Noise Optimization

- Next, define effective noise resistances (conductances):

$$R_n \equiv \frac{\overline{e_n^2}}{4kT\Delta f}, \quad G_u \equiv \frac{\overline{i_u^2}}{4kT\Delta f}, \quad G_S \equiv \frac{\overline{i_S^2}}{4kT\Delta f}$$

- Also:

- $Y_c = G_c + jB_c$

- $Y_s = G_s + jB_s$

- Finally obtain:

$$F = 1 + \frac{G_u}{G_s} + \frac{R_n}{G_s} \left[(G_s + G_c)^2 + (B_s + B_c)^2 \right]$$

Classic Two-Port Noise Optimization

- **Minimum F occurs when $B_s = -B_c = B_{opt}$ and**

$$G_s = \sqrt{\frac{G_u}{R_n} + G_c^2} = G_{opt}$$

- **Minimum F is**

$$F_{min} = 1 + 2R_n \left[\sqrt{\frac{G_u}{R_n} + G_c^2} + G_c^2 \right]$$

- **In general,**

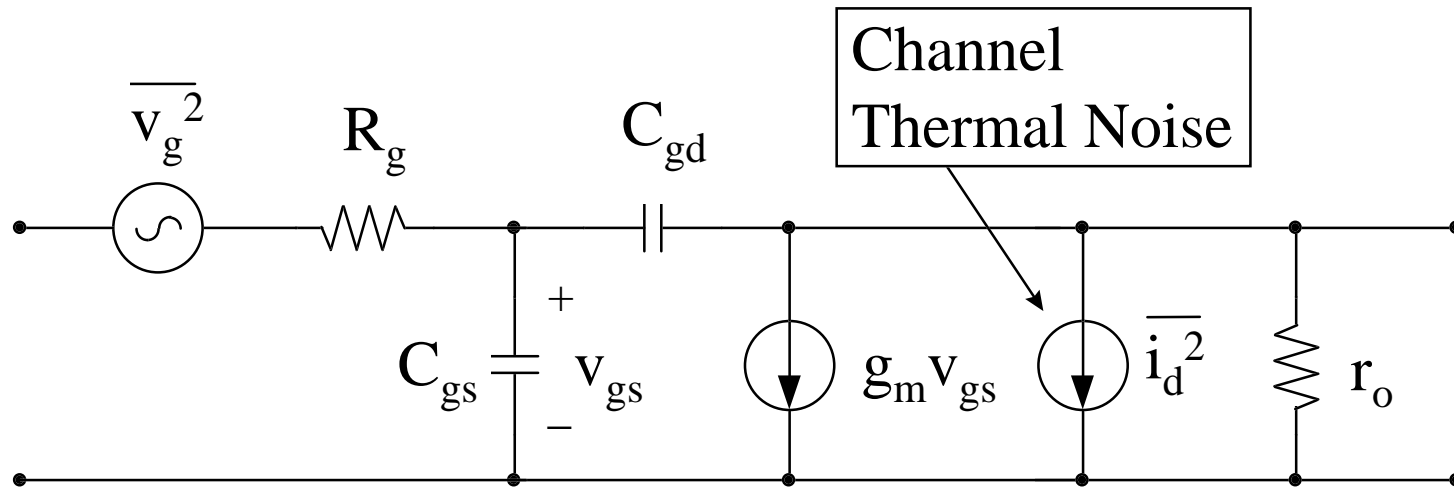
$$F = F_{min} + \frac{R_n}{G_s} \left[\left(G_s - G_{opt} \right)^2 + \left(B_s - B_{opt} \right)^2 \right]$$

- **Thus, contours of constant noise figure are circles centered about (G_{opt}, B_{opt}) in the admittance or Smith plane**

Classic Two-Port Noise Optimization

- ❑ **Source admittance for optimum noise match does not generally have any relation to the conditions for optimum power gain**
 - ❑ **Possible to have great noise figure and little or no gain**
 - ❑ **Possible to have great noise figure and a poor impedance match**
- ❑ **Classical noise optimization also does not consider power consumption directly**
 - ❑ **Modified approach required to balance all parameters of practical interest**

Simple CMOS Noise Model



- Channel thermal noise is dominant.

$$\overline{i_d^2} = 4kTB\gamma g_{d0}$$

- Gate resistance minimized by good layout.

Channel Thermal Noise

- Current HSPICE Implementation:

$$\overline{i_d^2} = \frac{8}{3} kTB g_m \quad (\text{NLEV} < 3)$$

$$\overline{i_d^2} = \frac{8}{3} kTB \cdot K' (V_{gs} - V_T) \frac{1 + a + a^2}{1 + a} GDSNOI \quad (\text{NLEV} = 3)$$

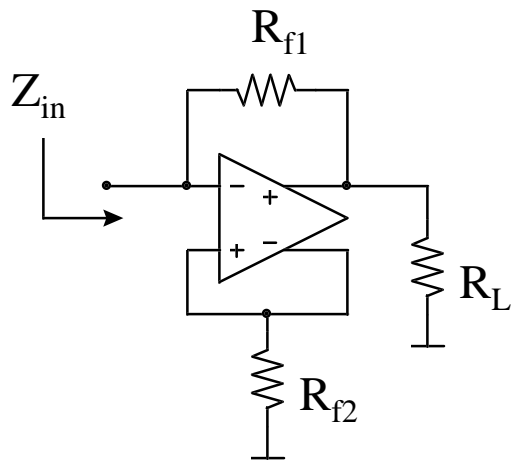
$$a = 1 - \frac{V_{ds}}{V_{dsat}}$$

- BSIM-3 Implementation:

$$\overline{i_d^2} = \frac{4kT\mu_{eff}}{L_{eff}^2} |Q_{inv}|$$



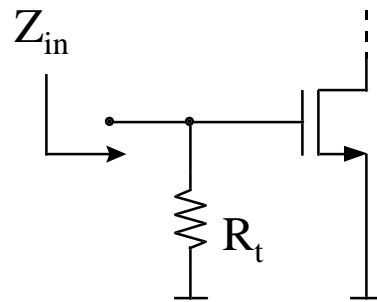
How To Get 50Ω



Dual Feedback

$$Z_{in} = \sqrt{R_{f1} R_{f2}}$$

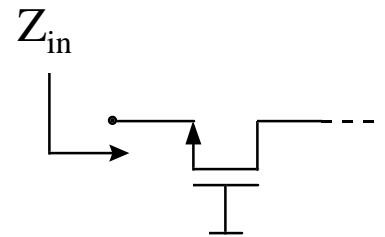
Need high gain.
Stability problems.



Resistive Termination

$$Z_{in} = R_t$$

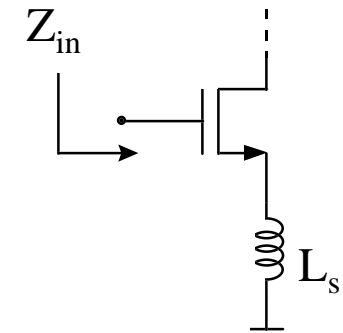
Poor NF.



$1/g_m$ Termination

$$Z_{in} = \frac{1}{g_m}$$

NF > 3dB
($\gamma > 1$)



Inductive Degeneration

$$\text{Re}[Z_{in}] = \frac{g_m}{C_{gs}} L_s$$

Narrowband.

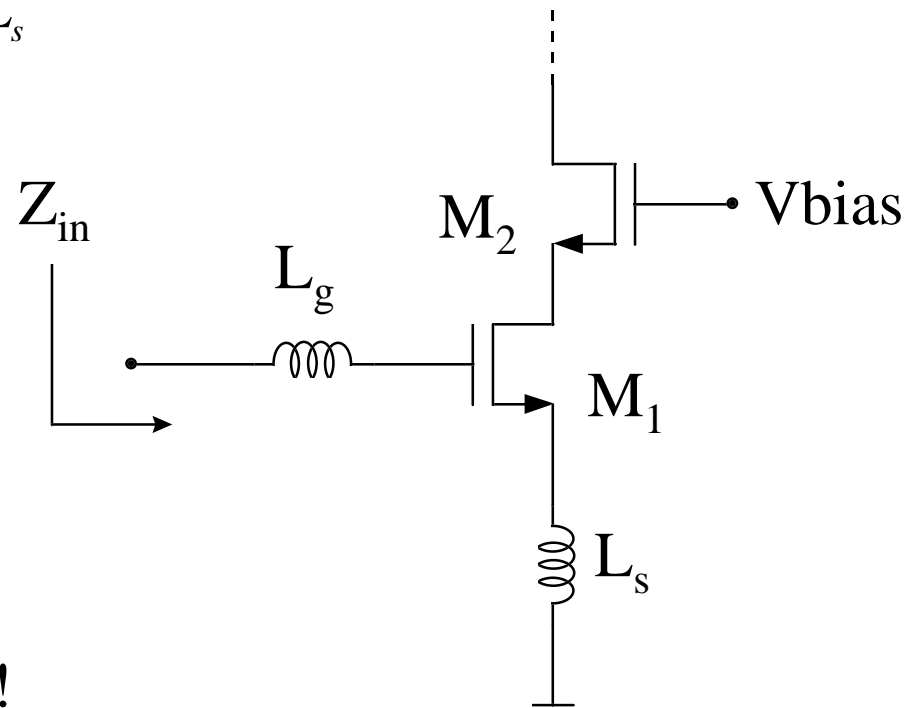


LNA Input Stage

$$Z_{in} = s(L_s + L_g) + \frac{1}{sC_{gs}} + \left(\frac{g_m}{C_{gs}}\right)L_s \approx \omega_T L_s$$

$$\begin{aligned} G_{m,eff} &= g_{m1} Q_{in} = \frac{g_{m1}}{\omega C_{gs} (R_s + \omega_T L_s)} \\ &= \frac{\omega_T}{\omega R_s \left(1 + \frac{\omega_T L_s}{R_s}\right)} = \frac{\omega_T}{2\omega R_s} \end{aligned}$$

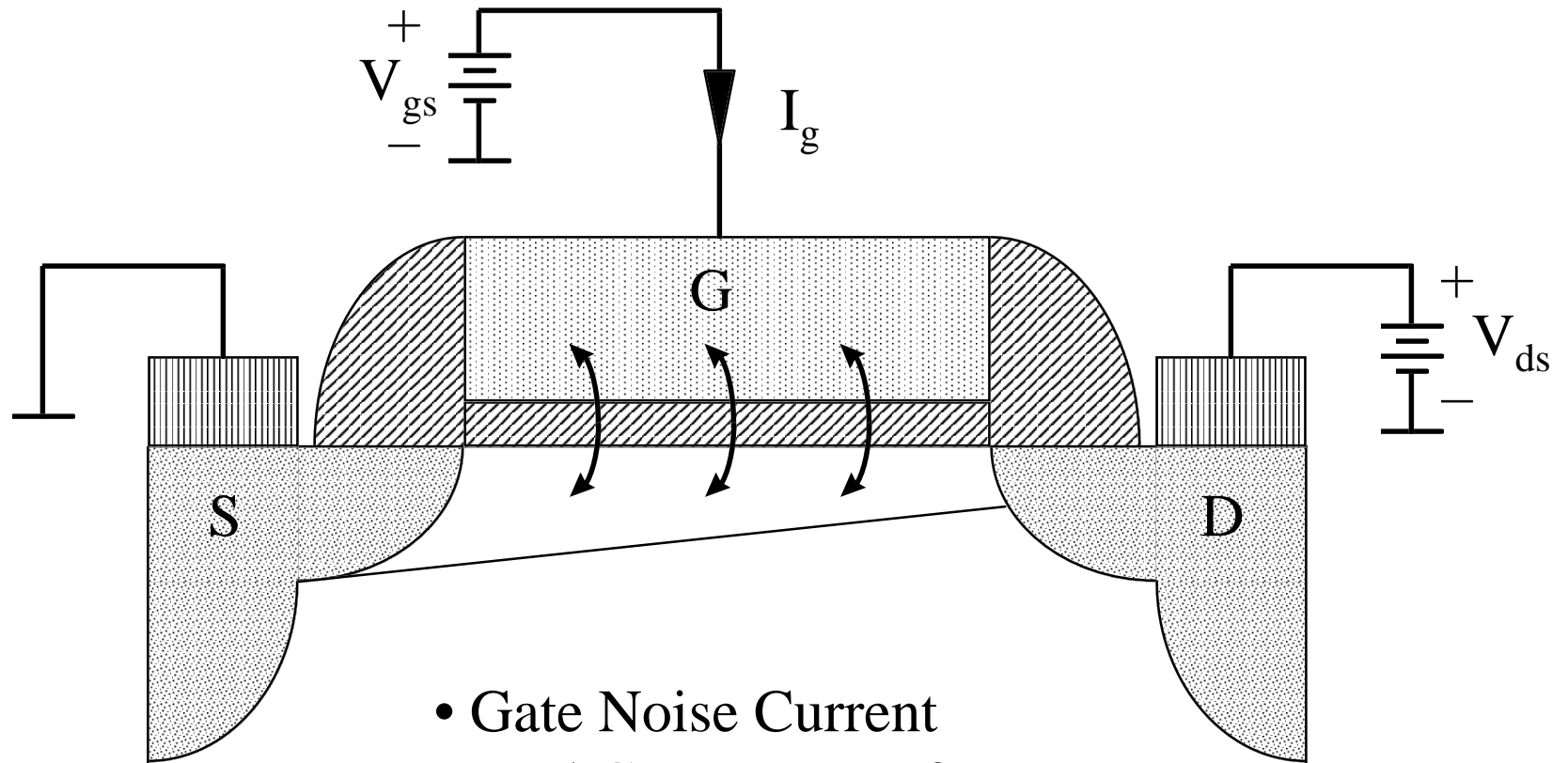
Note: $G_{m,eff}$ is independent of g_{m1} !



LNA Input Stage: Some Observations

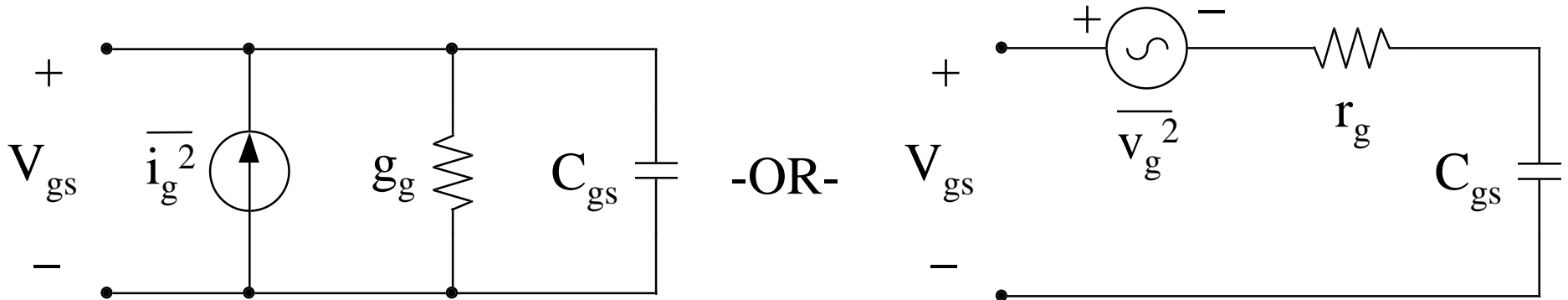
- ❑ **As noted, overall stage transconductance is independent of device g_m if resonant frequency and current density are held constant.**
 - ❑ **Theoretically, may use arbitrarily narrow devices and still obtain the desired transconductance.**
- ❑ **If drain current noise were the only noise source, narrower devices would lead to monotonically decreasing noise.**
- ❑ **Since gain is fixed, noise figure approaches 0dB as device narrows. Power dissipation would also approach zero.**
- ❑ **Absurd conclusion of zero dB NF, zero power dissipation and nonzero gain should make one suspect that something is missing from the foregoing.**

Induced Gate Effects



- Gate Noise Current
- Real Component of Z_g

Equivalent Gate Circuit



$$\overline{i_g^2} = 4kTB\delta g_g \quad g_g = \frac{1}{5} \frac{\omega^2 C_{gs}^2}{g_{d0}}$$

$$\overline{v_g^2} = 4kTB\delta r_g \quad r_g = \frac{1}{5g_{d0}}$$

“Blue” Noise

- δ ($\sim 4/3$) modified by hot electron effects
- $\overline{i_g^2}$ partially correlated with $\overline{i_d^2}$ ($c = 0.395j$)
- $\overline{i_g^2}$ and g_g not modeled in HSPICE

“White” Noise



MOSFET Two-Port Noise Parameters

$$\square \quad B_{opt} = -B_c = -\omega C_{gs} \left(1 + \alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right)$$

\square **B_{opt} is inductive, except for frequency behavior. Difficult to provide this behavior over a large bandwidth.**

$$\square \quad G_{opt} = \sqrt{\frac{G_u}{R_n} + G_c^2} = \alpha \omega C_{gs} \sqrt{\frac{\delta}{5\gamma} (1 - |c|^2)}$$

$$\square \quad F_{min} = 1 + 2R_n [G_{opt} + G_c] \approx 1 + \frac{2}{\sqrt{5}} \frac{\omega}{\omega_T} \sqrt{\gamma \delta (1 - |c|^2)}$$

\square **Note that $F_{min} = 0\text{dB}$ if gate and drain noise were fully correlated. *The mere presence of noise sources does not necessarily imply nonzero NF.***

MOSFET Two-Port Noise Parameters

- Consider only drain and induced gate current noise. Then, the following two-port parameters apply:

Parameter	Expression
G_c	≈ 0
B_c	$\omega C_{gs} \left(1 + \alpha c \sqrt{\frac{\delta}{5\gamma}} \right)$
R_n	$\frac{\gamma g_{d0}}{2 g_m} = \frac{\gamma}{\alpha} \cdot \frac{1}{g_m}$
G_u	$\frac{\delta \omega^2 C_{gs}^2 (1 - c ^2)}{5 g_{d0}}$

MOSFET Two-Port Noise Parameters

- ❑ Let's now compile a short table of F_{min} values:

$\frac{g_m}{\omega C_{gs}}$	F_{min} (dB)
20	0.5
15	0.6
10	0.9
5	1.6

- ❑ Numbers pessimistically assume that hot electron effects triple the mean-square noise densities.
- ❑ Even with such effects, achievable noise figures are very good.
- ❑ Question: How can these values be approached in practice?

Second-Order Noise Sources

- ❑ Practical NF values are affected by series gate resistance and epi noise.
- ❑ F is increased by R_g/R_s , so just 10Ω by itself sets a lower NF bound of 0.8dB in a 50Ω system.
 - ❑ Must use multi-fingered devices ($R_{\text{finger}} = R_{\text{SH}}W_{\text{finger}}/3L$).
 - ❑ Cannot use planar spiral inductors in gate circuit if best NF is to be achieved (NF typically > 2 -3dB).
- ❑ Thermal noise of substrate (epi) resistance modulates the back gate, giving rise to additional drain current noise:

$$\frac{i_{nd}^2}{\Delta f} = 4kT \left(\gamma g_{d0} + g_{mb}^2 R_{\text{epi}} \right) = 4kT g_{d0} \left(\gamma + \frac{g_{mb}^2 R_{\text{epi}}}{g_{d0}} \right)$$

Second-Order Noise Sources

- ❑ **Effect of epi noise is equivalent to an increase in γ :**

$$\gamma_{eff} = \gamma + \frac{g_{mb}^2 R_{epi}}{g_{d0}}$$

- ❑ **One may compute that, typically, epi noise increases γ by $\sim 10\%$, an amount smaller than the uncertainty in γ itself.**
- ❑ **Epi noise also contributes to equivalent input current noise, but this is fully correlated with the drain noise.**
 - ❑ **Again, fundamental NF limits are set by the *uncorrelated* gate and drain noise components.**

Narrowband LNA

- ❑ Choose inductive source degeneration to produce desired real part:

$$L_S \approx \frac{R_S \cdot \left[1 + 2 \left(C_{gd} / C_{gs} \right) \right]}{\omega_T}$$

- ❑ Equation assumes a cascode stack with equal-sized devices
- ❑ Choose sum of gate and source degenerating inductances either to resonate with C_{gs} or to provide a susceptance equal to B_{opt} .
 - ❑ First choice maximizes gain, second choice minimizes NF. Difference is small because $B_{opt} \approx \omega C_{gs}$.
- ❑ Note that classic noise optimization says nothing about power dissipation, nor anything about how to select device width.

Power-Constrained Noise Optimization

- ❑ **Good approximation: Select device width roughly equal to $(500\mu\text{m}\cdot\text{GHz})/f_0$ (for a 50Ω system).**
- ❑ **Adjust bias to obtain desired power dissipation.**
 - ❑ **Keep $V_{DS}-V_{DSAT}$ as small as practical to minimize hot-electron effects (say, under half a volt or so).**
- ❑ **For equal-sized cascoding and main devices, continue to select source degeneration inductance according to:**

$$L_S \approx \frac{R_S \cdot \left[1 + 2 \left(C_{gd} / C_{gs} \right) \right]}{\omega_T}$$

- ❑ **Add gate inductance to bring input to resonance.**
- ❑ **Noise factor bound is $1 + 2.4(\gamma/\alpha)(\omega/\omega_T)$, so scaling continues to help directly.**

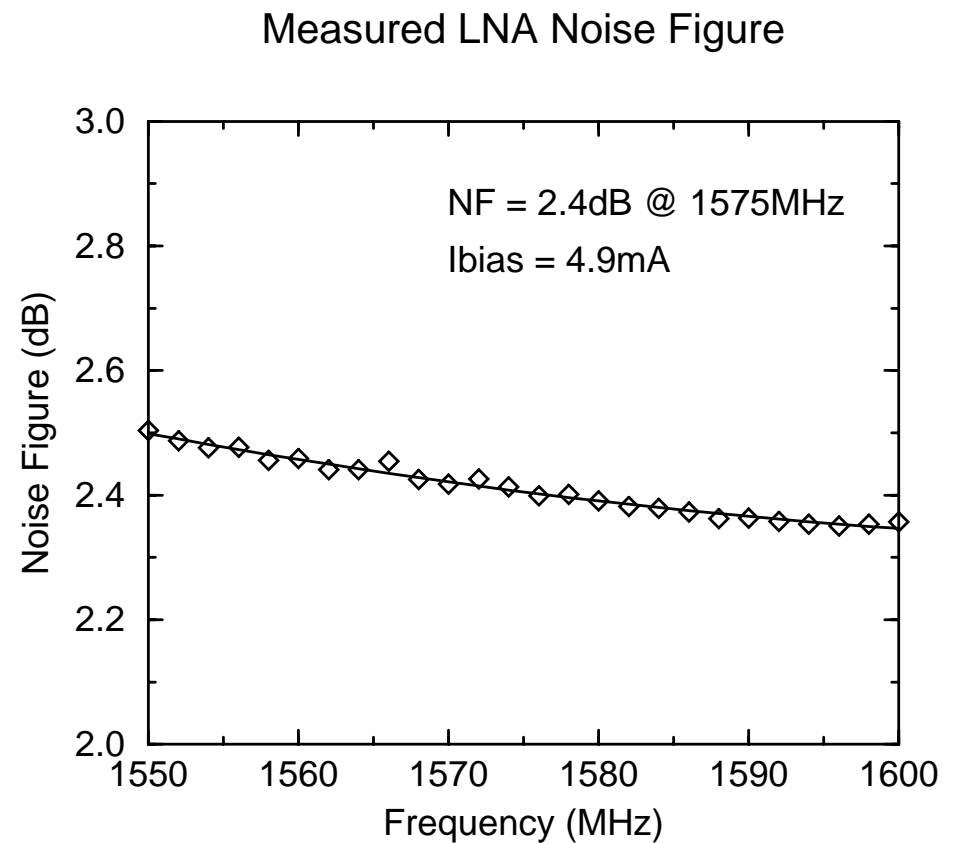
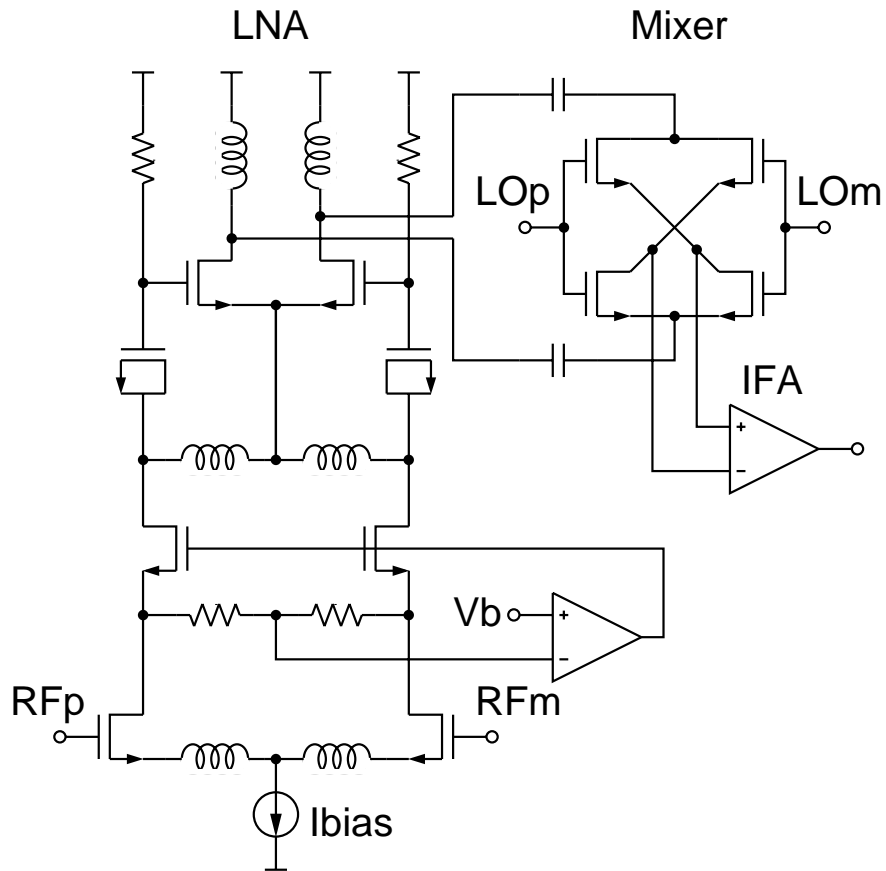
Experimental Results: Devices

- ❑ For 0.5 μm technology (drawn), $\text{NF}_{\text{min}} \approx 1.0\text{dB}$ @ 2mA, 1GHz.
 - ❑ NF_{min} decreases to $\approx 0.7\text{dB}$ @ high I_{D} .
 - ❑ NF_{min} increases to $\approx 1.3\text{dB}$ @ 2GHz @ high I_{D} .
 - ❑ NF_{min} still below 2dB @ 400 μA , 1GHz.
- ❑ These values apply to a single device without regard for input impedance.
 - ❑ Practical NF_{min} values are perhaps 0.5dB to 1dB higher.
- ❑ Contrary to expectations, no increase in NF_{min} is observed in these devices as V_{DS} increases in saturation.
 - ❑ Drain engineering possibly responsible (G. Klimovitch et al., 1997).

Experimental Results: Circuits

- ❑ **Single-ended versions consume half the power for a given NF than differential versions, but:**
 - ❑ **No rejection of common-mode noise.**
 - ❑ **Very sensitive to parasitics, particularly inductances in the source lead of the main transistor.**
- ❑ **Differential versions are relatively insensitive to hard-to-model and hard-to-control parasitics.**
 - ❑ **Attractive for high-volume production.**
 - ❑ **Common-mode rejection highly desirable for mixed-signal environments.**

CIRCUITS: LNA / MIXER



Shahani, Shaeffer and Lee, "A 12mW Wide Dynamic Range CMOS GPS Receiver," ISSCC 1997

Experimental Results: Circuits

- ❑ **Series gate inductance provided by bondwires to avoid inevitable NF degradation associated with spiral inductors.**
 - ❑ **Difficult to obtain accurate value without trimming, but repeatability with automated die attach and bonding equipment is very good.**
 - ❑ **Input Q is generally 3-5, so LNA is somewhat forgiving of moderate element value variation.**
- ❑ **Measured and simulated NF agree to within 0.2dB.**
- ❑ **$S_{11} < -15\text{dB}$.**
- ❑ **Receiver IIP3 $> -16\text{dBm}$ (measurement confounded by linearity limitation of subsequent receiver stages).**
 - ❑ **IIP3 $> -6\text{dBm}$ for LNA itself (simulated).**

Summary and Conclusions

- ❑ **CMOS devices are capable of excellent noise performance in the low-GHz frequency range.**
 - ❑ **Noise performance will continue to improve, despite fears that hot-electron effects will nullify the benefits of scaling.**
- ❑ **Inductively-degenerated LNA architecture simultaneously provides near-optimum gain and NF.**
 - ❑ **Proper device width is important, also.**
- ❑ **At under 10mW dissipation, practical single-ended amplifier noise figures of $\sim 1.5\text{dB}$ at 1GHz are achievable with $0.5\mu\text{m}$ technology.**
- ❑ **Short-channel effects improve linearity, so dynamic range per power will improve with scaling.**
- ❑ **Epi and gate resistance noise effects are minor, or can be made so.**

Acknowledgments

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