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# Narrowband CMOS RF Low-Noise Amplifiers

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# Outline

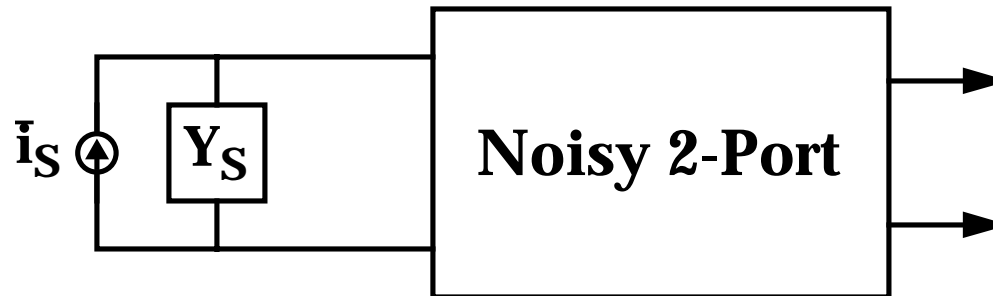
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- ❑ **A brief review of classic two-port noise optimization**
  - ❑ **Conditions for minimum noise figure**
  - ❑ **The fundamental importance of correlations**
- ❑ **MOSFET noise models in the short-channel regime**
  - ❑ **Equivalent two-port noise generators**
  - ❑ **Second-order noise sources**
- ❑ **Power constrained noise optimization**
- ❑ **Experimental results on devices and circuits**
- ❑ **Summary and conclusions**

# Classic Two-Port Noise Optimization

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- ❑ Consider noise in an arbitrary (but linear) system:

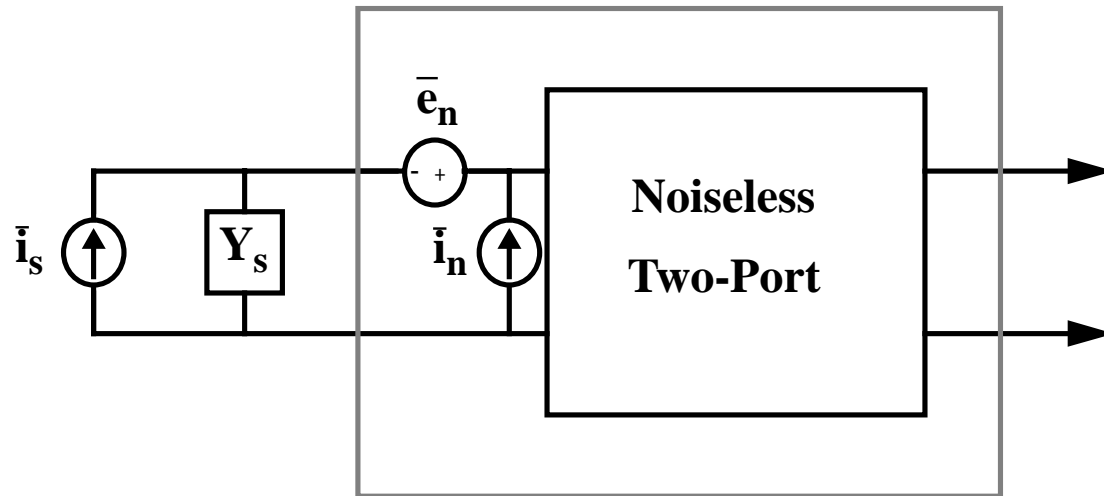


- ❑ Thermal noise of source represented by  $\bar{i}_S$
- ❑ Source admittance is  $Y_S$

# Classic Two-Port Noise Optimization

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- ❑ The noisy two-port may be modeled as follows:



- ❑ In general, the external noise sources will be partially *correlated*
  - ❑ Correlations arise because an internal noise source may contribute to both  $\bar{i}_n$  and  $\bar{e}_n$  in general
  - ❑ Correlations have strong implications for noise performance

# Classic Two-Port Noise Optimization

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- ❑ Noise factor,  $F$ , is defined as the ratio of the total output noise power divided by that part of the output noise power due to the input source, when source is at 290K
- ❑ Therefore:

$$F = \frac{\overline{i_s^2} + \overline{|i_n + Y_s e_n|^2}}{\overline{i_s^2}}$$

- ❑ Let noise current  $i_n$  be expressed as sum of two terms
  - ❑ First term,  $i_u$ , is fully uncorrelated with noise voltage  $e_n$ .  
Other term,  $i_c$ , is fully correlated with  $e_n$ .

# Classic Two-Port Noise Optimization

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- Since  $i_c$  is correlated with  $e_n$ , we may write one as proportional to the other:

$$i_c = Y_c e_n$$

- Note that  $Y_c$  has the dimensions of an admittance
  - Correlation admittance is a mathematical construct, and is not what one measures with an impedance meter
- Re-write  $F$  as

$$F = 1 + \frac{\overline{|i_n + Y_S e_n|^2}}{\overline{i_S^2}} = 1 + \frac{\overline{i_u^2} + \overline{|Y_c + Y_S|^2 e_n^2}}{\overline{i_S^2}}$$

# Classic Two-Port Noise Optimization

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- Next, define effective noise resistances (conductances):

$$R_n \equiv \frac{\overline{e_n^2}}{4kT\Delta f}, \quad G_u \equiv \frac{\overline{i_u^2}}{4kT\Delta f}, \quad G_S \equiv \frac{\overline{i_S^2}}{4kT\Delta f}$$

- Also:

- $Y_c = G_c + jB_c$

- $Y_s = G_s + jB_s$

- Finally obtain:

$$F = 1 + \frac{G_u}{G_s} + \frac{R_n}{G_s} \left[ (G_s + G_c)^2 + (B_s + B_c)^2 \right]$$

# Classic Two-Port Noise Optimization

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- Minimum  $F$  occurs when  $B_s = -B_c = B_{opt}$  and

$$G_s = \sqrt{\frac{G_u}{R_n} + G_c^2} = G_{opt}$$

- Minimum  $F$  is

$$F_{min} = 1 + 2R_n \left[ \sqrt{\frac{G_u}{R_n} + G_c^2} + G_c^2 \right]$$

- In general,

$$F = F_{min} + \frac{R_n}{G_s} \left[ \left( G_s - G_{opt} \right)^2 + \left( B_s - B_{opt} \right)^2 \right]$$

- Thus, contours of constant noise figure are circles centered about  $(G_{opt}, B_{opt})$  in the admittance or Smith plane

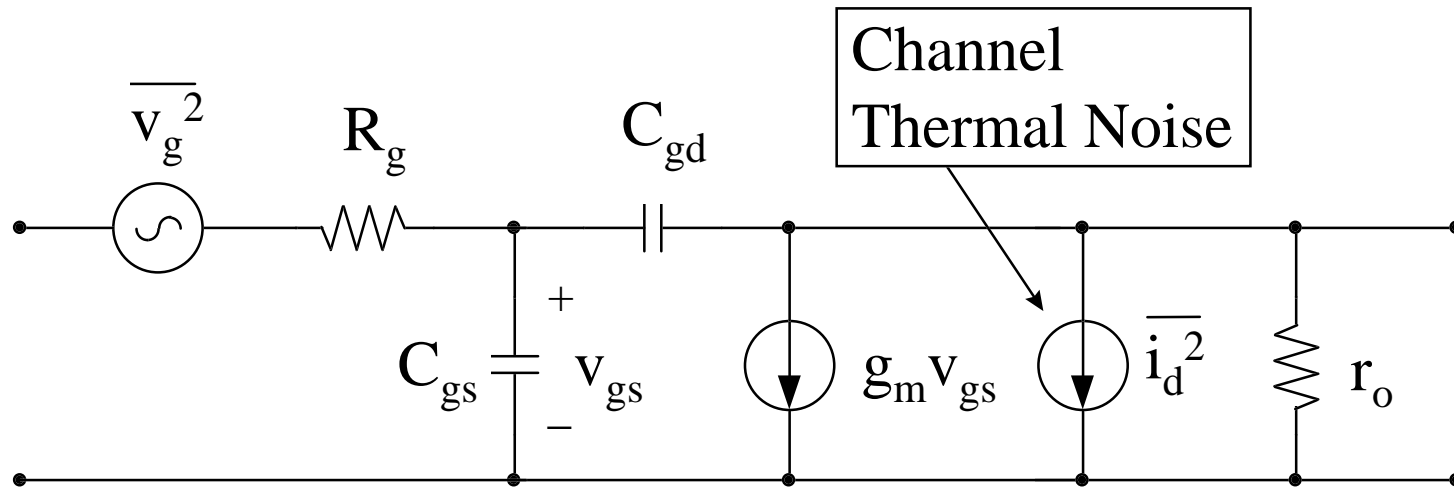


# Classic Two-Port Noise Optimization

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- ❑ **Source admittance for optimum noise match does not generally have any relation to the conditions for optimum power gain**
  - ❑ **Possible to have great noise figure and little or no gain**
  - ❑ **Possible to have great noise figure and a poor impedance match**
- ❑ **Classical noise optimization also does not consider power consumption directly**
  - ❑ **Modified approach required to balance all parameters of practical interest**

# Simple CMOS Noise Model



- Channel thermal noise is dominant.

$$\overline{i_d^2} = 4kTB\gamma g_{d0}$$

- Gate resistance minimized by good layout.

# Channel Thermal Noise

- Current HSPICE Implementation:

$$\overline{i_d^2} = \frac{8}{3} kTB g_m \quad (\text{NLEV} < 3)$$

$$\overline{i_d^2} = \frac{8}{3} kTB \cdot K' (V_{gs} - V_T) \frac{1 + a + a^2}{1 + a} GDSNOI \quad (\text{NLEV} = 3)$$

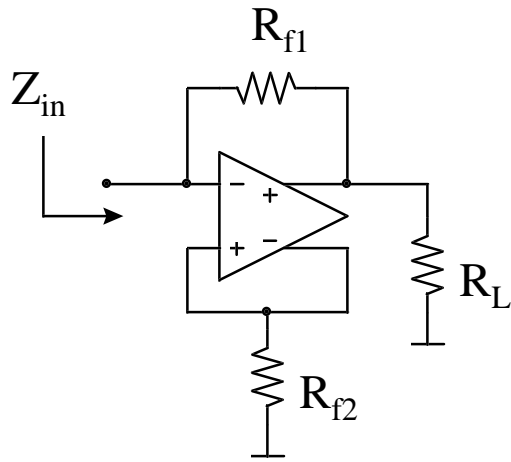
$$a = 1 - \frac{V_{ds}}{V_{dsat}}$$

- BSIM-3 Implementation:

$$\overline{i_d^2} = \frac{4kT\mu_{eff}}{L_{eff}^2} |Q_{inv}|$$



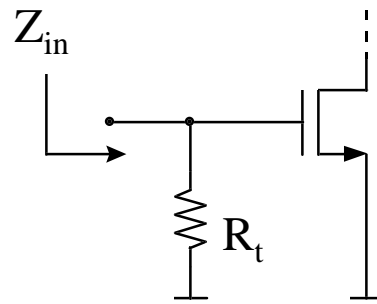
# How To Get 50Ω



Dual Feedback

$$Z_{in} = \sqrt{R_{f1} R_{f2}}$$

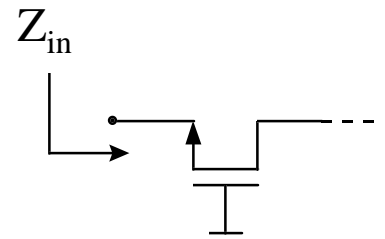
Need high gain.  
Stability problems.



Resistive Termination

$$Z_{in} = R_t$$

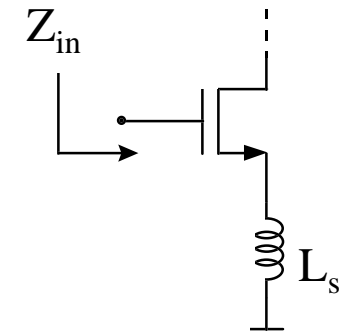
Poor NF.



$1/g_m$  Termination

$$Z_{in} = \frac{1}{g_m}$$

NF > 3dB  
( $\gamma > 1$ )



Inductive Degeneration

$$\text{Re}[Z_{in}] = \frac{g_m}{C_{gs}} L_s$$

Narrowband.

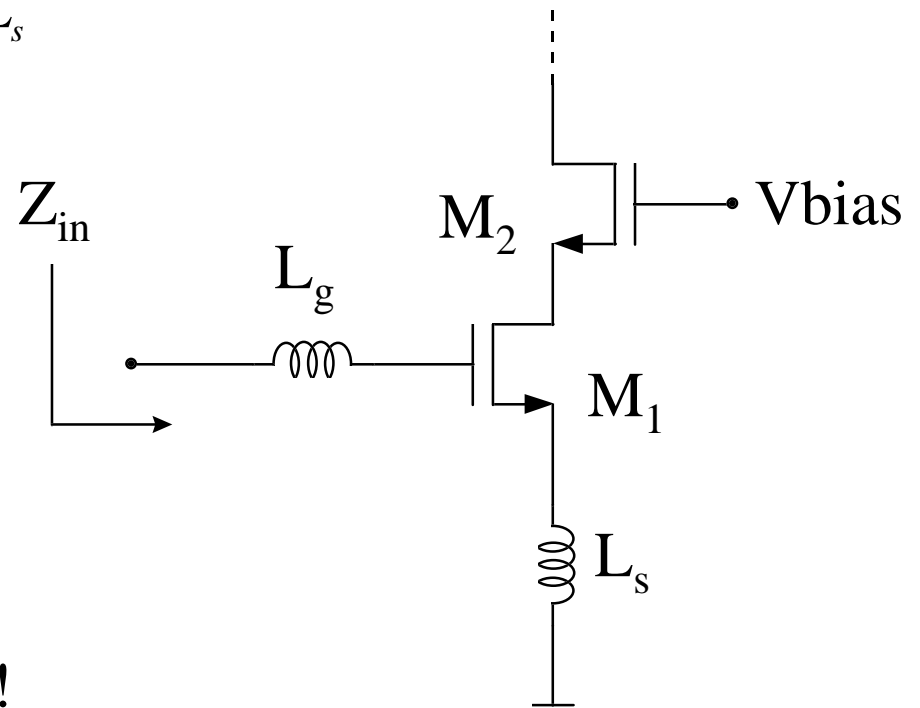


# LNA Input Stage

$$Z_{in} = s(L_s + L_g) + \frac{1}{sC_{gs}} + \left(\frac{g_m}{C_{gs}}\right)L_s \approx \omega_T L_s$$

$$\begin{aligned} G_{m,eff} &= g_{m1} Q_{in} = \frac{g_{m1}}{\omega C_{gs} (R_s + \omega_T L_s)} \\ &= \frac{\omega_T}{\omega R_s \left(1 + \frac{\omega_T L_s}{R_s}\right)} = \frac{\omega_T}{2\omega R_s} \end{aligned}$$

Note:  $G_{m,eff}$  is independent of  $g_{m1}$ !

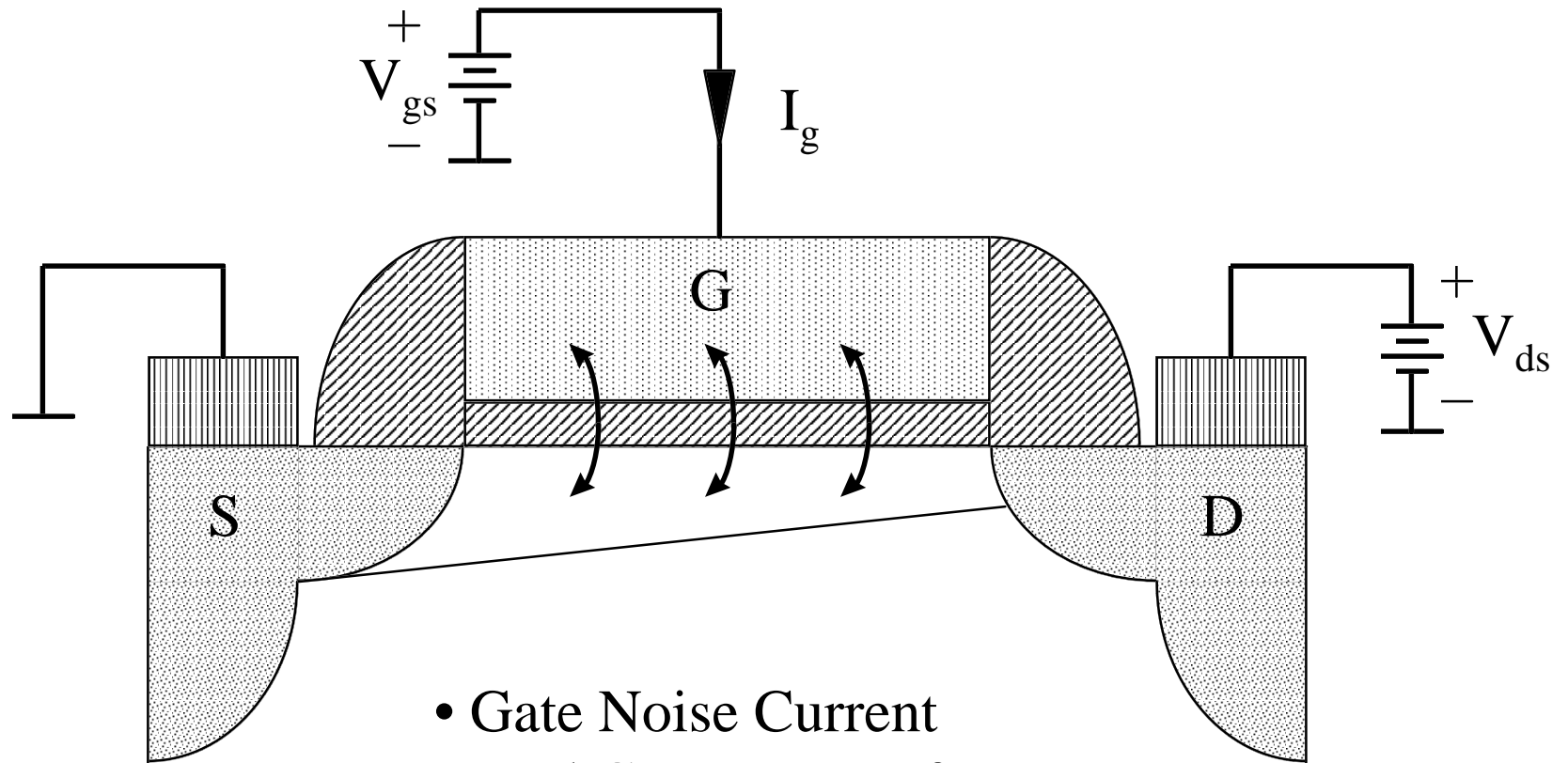


# LNA Input Stage: Some Observations

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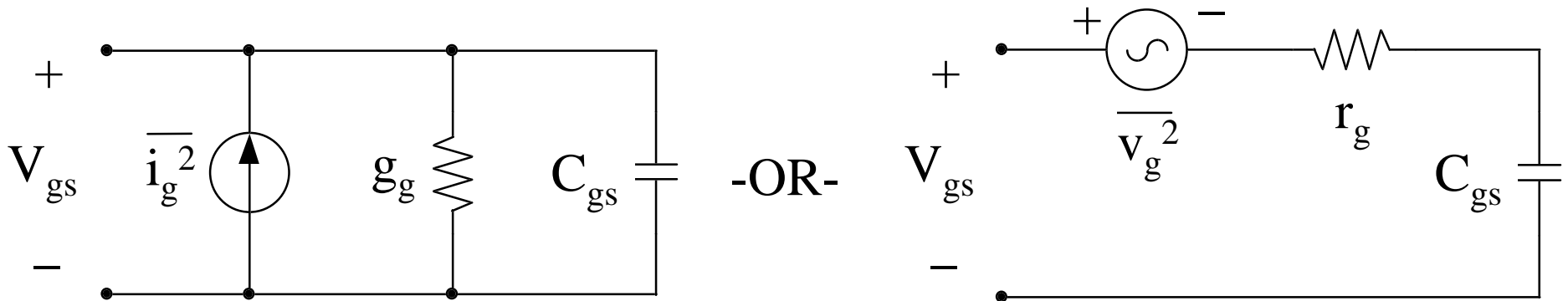
- ❑ **As noted, overall stage transconductance is independent of device  $g_m$  if resonant frequency and current density are held constant.**
  - ❑ **Theoretically, may use arbitrarily narrow devices and still obtain the desired transconductance.**
- ❑ **If drain current noise were the only noise source, narrower devices would lead to monotonically decreasing noise.**
- ❑ **Since gain is fixed, noise figure approaches 0dB as device narrows. Power dissipation would also approach zero.**
- ❑ **Absurd conclusion of zero dB NF, zero power dissipation and nonzero gain should make one suspect that something is missing from the foregoing.**

# Induced Gate Effects



- Gate Noise Current
- Real Component of  $Z_g$

# Equivalent Gate Circuit



$$\overline{i_g^2} = 4kTB\delta g_g \quad g_g = \frac{1}{5} \frac{\omega^2 C_{gs}^2}{g_{d0}}$$

$$\overline{v_g^2} = 4kTB\delta r_g \quad r_g = \frac{1}{5g_{d0}}$$

“Blue” Noise

“White” Noise

- $\delta$  ( $\sim 4/3$ ) modified by hot electron effects
- $\overline{i_g^2}$  partially correlated with  $\overline{i_d^2}$  ( $c = 0.395j$ )
- $\overline{i_g^2}$  and  $g_g$  not modeled in HSPICE





# MOSFET Two-Port Noise Parameters

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$$\square \quad B_{opt} = -B_c = -\omega C_{gs} \left( 1 + \alpha |c| \sqrt{\frac{\delta}{5\gamma}} \right)$$

$\square$   $B_{opt}$  is inductive, except for frequency behavior. Difficult to provide this behavior over a large bandwidth.

$$\square \quad G_{opt} = \sqrt{\frac{G_u}{R_n} + G_c^2} = \alpha \omega C_{gs} \sqrt{\frac{\delta}{5\gamma} (1 - |c|^2)}$$

$$\square \quad F_{min} = 1 + 2R_n [G_{opt} + G_c] \approx 1 + \frac{2}{\sqrt{5}} \frac{\omega}{\omega_T} \sqrt{\gamma \delta (1 - |c|^2)}$$

$\square$  Note that  $F_{min} = 0\text{dB}$  if gate and drain noise were fully correlated. *The mere presence of noise sources does not necessarily imply nonzero NF.*

# MOSFET Two-Port Noise Parameters

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- ❑ Consider only drain and induced gate current noise. Then, the following two-port parameters apply:

Parameter	Expression
$G_c$	$\approx 0$
$B_c$	$\omega C_{gs} \left( 1 + \alpha  c  \sqrt{\frac{\delta}{5\gamma}} \right)$
$R_n$	$\frac{\gamma g_{d0}}{2 g_m} = \frac{\gamma}{\alpha} \cdot \frac{1}{g_m}$
$G_u$	$\frac{\delta \omega^2 C_{gs}^2 (1 -  c ^2)}{5 g_{d0}}$

# MOSFET Two-Port Noise Parameters

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- ❑ Let's now compile a short table of  $F_{min}$  values:

$\frac{g_m}{\omega C_{gs}}$	$F_{min}$ (dB)
20	0.5
15	0.6
10	0.9
5	1.6

- ❑ Numbers pessimistically assume that hot electron effects triple the mean-square noise densities.
- ❑ Even with such effects, achievable noise figures are very good.
- ❑ Question: How can these values be approached in practice?

# Second-Order Noise Sources

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- ❑ Practical NF values are affected by series gate resistance and epi noise.
- ❑ F is increased by  $R_g/R_s$ , so just  $10\Omega$  by itself sets a lower NF bound of 0.8dB in a  $50\Omega$  system.
  - ❑ Must use multi-fingered devices ( $R_{\text{finger}} = R_{\text{SH}}W_{\text{finger}}/3L$ ).
  - ❑ Cannot use planar spiral inductors in gate circuit if best NF is to be achieved (NF typically  $> 2\text{-}3\text{dB}$ ).
- ❑ Thermal noise of substrate (epi) resistance modulates the back gate, giving rise to additional drain current noise:

$$\frac{i_{nd}^2}{\Delta f} = 4kT \left( \gamma g_{d0} + g_{mb}^2 R_{\text{epi}} \right) = 4kT g_{d0} \left( \gamma + \frac{g_{mb}^2 R_{\text{epi}}}{g_{d0}} \right)$$

# Second-Order Noise Sources

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- ❑ Effect of epi noise is equivalent to an increase in  $\gamma$ :

$$\gamma_{eff} = \gamma + \frac{g_{mb}^2 R_{epi}}{g_{d0}}$$

- ❑ One may compute that, typically, epi noise increases  $\gamma$  by  $\sim 10\%$ , an amount smaller than the uncertainty in  $\gamma$  itself.
- ❑ Epi noise also contributes to equivalent input current noise, but this is fully correlated with the drain noise.
  - ❑ Again, fundamental NF limits are set by the *uncorrelated* gate and drain noise components.

# Narrowband LNA

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- ❑ Choose inductive source degeneration to produce desired real part:

$$L_S \approx \frac{R_S \cdot \left[ 1 + 2 \left( C_{gd} / C_{gs} \right) \right]}{\omega_T}$$

- ❑ Equation assumes a cascode stack with equal-sized devices
- ❑ Choose sum of gate and source degenerating inductances either to resonate with  $C_{gs}$  or to provide a susceptance equal to  $B_{opt}$ .
  - ❑ First choice maximizes gain, second choice minimizes NF. Difference is small because  $B_{opt} \approx \omega C_{gs}$ .
- ❑ Note that classic noise optimization says nothing about power dissipation, nor anything about how to select device width.

# Power-Constrained Noise Optimization

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- ❑ **Good approximation: Select device width roughly equal to  $(500\mu\text{m}\cdot\text{GHz})/f_0$  (for a  $50\Omega$  system).**
- ❑ **Adjust bias to obtain desired power dissipation.**
  - ❑ **Keep  $V_{DS}-V_{DSAT}$  as small as practical to minimize hot-electron effects (say, under half a volt or so).**
- ❑ **For equal-sized cascoding and main devices, continue to select source degeneration inductance according to:**

$$L_S \approx \frac{R_S \cdot \left[ 1 + 2 \left( C_{gd} / C_{gs} \right) \right]}{\omega_T}$$

- ❑ **Add gate inductance to bring input to resonance.**
- ❑ **Noise factor bound is  $1 + 2.4(\gamma/\alpha)(\omega/\omega_T)$ , so scaling continues to help directly.**

# Experimental Results: Devices

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- ❑ For 0.5 $\mu\text{m}$  technology (drawn),  $\text{NF}_{\text{min}} \approx 1.0\text{dB}$  @ 2mA, 1GHz.
  - ❑  $\text{NF}_{\text{min}}$  decreases to  $\approx 0.7\text{dB}$  @ high  $I_{\text{D}}$ .
  - ❑  $\text{NF}_{\text{min}}$  increases to  $\approx 1.3\text{dB}$  @ 2GHz @ high  $I_{\text{D}}$ .
  - ❑  $\text{NF}_{\text{min}}$  still below 2dB @ 400 $\mu\text{A}$ , 1GHz.
- ❑ These values apply to a single device without regard for input impedance.
  - ❑ Practical  $\text{NF}_{\text{min}}$  values are perhaps 0.5dB to 1dB higher.
- ❑ Contrary to expectations, no increase in  $\text{NF}_{\text{min}}$  is observed in these devices as  $V_{\text{DS}}$  increases in saturation.
  - ❑ Drain engineering possibly responsible (G. Klimovitch et al., 1997).

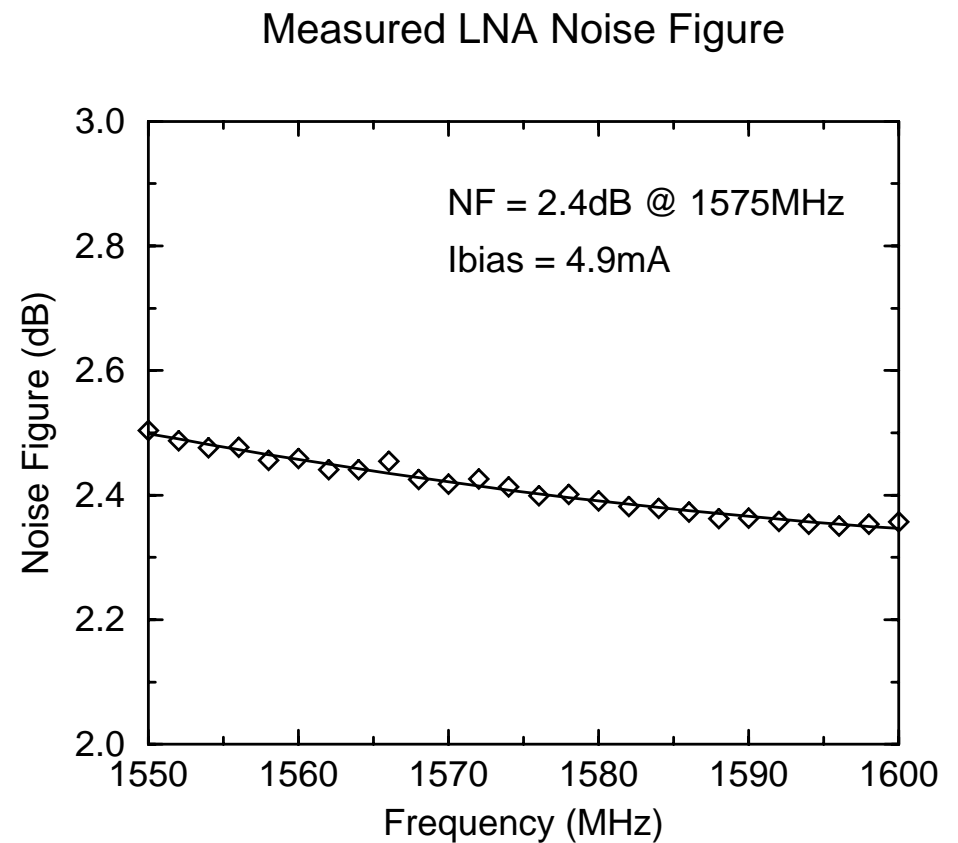
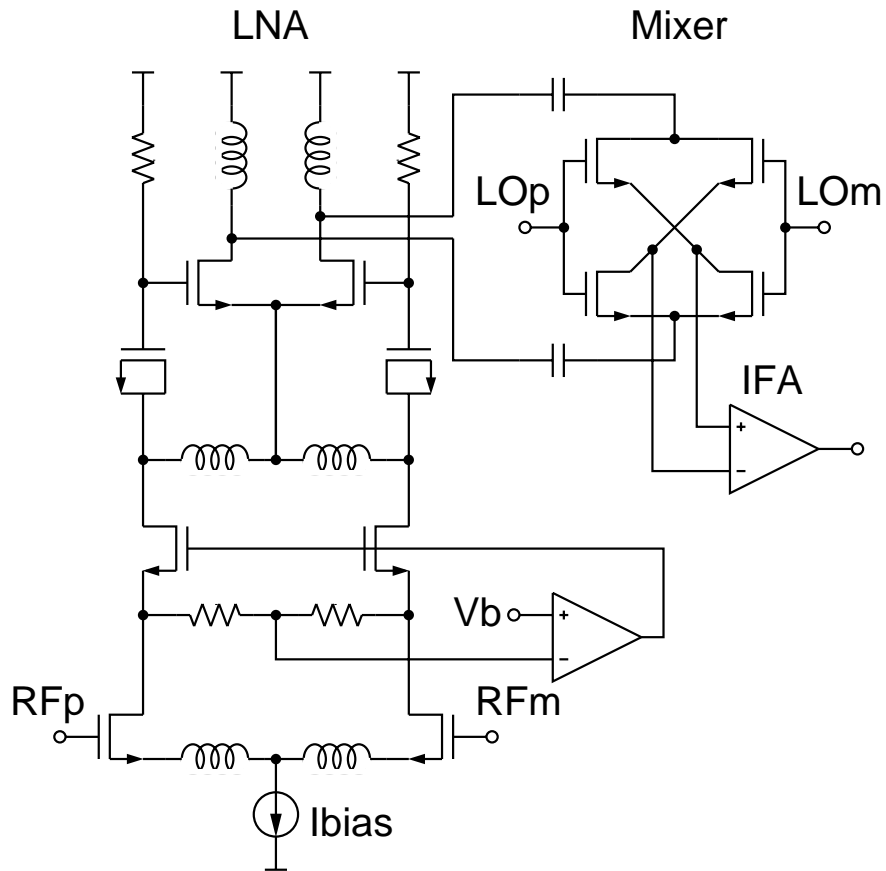


# Experimental Results: Circuits

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- ❑ **Single-ended versions consume half the power for a given NF than differential versions, but:**
  - ❑ **No rejection of common-mode noise.**
  - ❑ **Very sensitive to parasitics, particularly inductances in the source lead of the main transistor.**
- ❑ **Differential versions are relatively insensitive to hard-to-model and hard-to-control parasitics.**
  - ❑ **Attractive for high-volume production.**
  - ❑ **Common-mode rejection highly desirable for mixed-signal environments.**

# CIRCUITS: LNA / MIXER



Shahani, Shaeffer and Lee, "A 12mW Wide Dynamic Range CMOS GPS Receiver," ISSCC 1997

# Experimental Results: Circuits

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- ❑ Series gate inductance provided by bondwires to avoid inevitable NF degradation associated with spiral inductors.
  - ❑ Difficult to obtain accurate value without trimming, but repeatability with automated die attach and bonding equipment is very good.
  - ❑ Input  $Q$  is generally 3-5, so LNA is somewhat forgiving of moderate element value variation.
- ❑ Measured and simulated NF agree to within 0.2dB.
- ❑  $S_{11} < -15\text{dB}$ .
- ❑ Receiver IIP3  $> -16\text{dBm}$  (measurement confounded by linearity limitation of subsequent receiver stages).
  - ❑ IIP3  $> -6\text{dBm}$  for LNA itself (simulated).

# Summary and Conclusions

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- ❑ **CMOS devices are capable of excellent noise performance in the low-GHz frequency range.**
  - ❑ **Noise performance will continue to improve, despite fears that hot-electron effects will nullify the benefits of scaling.**
- ❑ **Inductively-degenerated LNA architecture simultaneously provides near-optimum gain and NF.**
  - ❑ **Proper device width is important, also.**
- ❑ **At under 10mW dissipation, practical single-ended amplifier noise figures of  $\sim 1.5\text{dB}$  at 1GHz are achievable with  $0.5\mu\text{m}$  technology.**
- ❑ **Short-channel effects improve linearity, so dynamic range per power will improve with scaling.**
- ❑ **Epi and gate resistance noise effects are minor, or can be made so.**

# Acknowledgments

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