CMOS Operational Amplifier Design and Optimization via Geometric Programming

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Main Idea

- Many op-amp design problems can be cast as a special type of **convex optimization problems**.
- Such problems can appear very difficult, but can be solved **very efficiently** by recently developed interior-point (IP) methods.

Outline

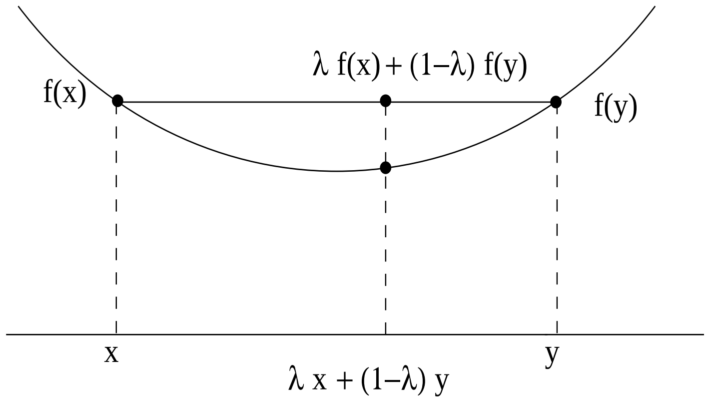
- Convex Optimization
- Posynomial Functions
- Geometric Programming
- Application to a Two-Stage Operational Amplifier
- Results
- Conclusions

Convex Optimization

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_1(x) \leq 0, \ldots, f_L(x) \leq 0, \quad Ax = b \end{array}$$

- $x \in \Re^n$ is optimization variable
- f_i are **convex**:

for $0 \leq \lambda \leq 1$,
$f_i(\lambda x + (1-\lambda)y)$
$\leq \lambda f_i(x) + (1-\lambda) f_i(y)$



Convex Optimization problems are fundamentally tractable

- computation time is small, grows gracefully with problem size and required accuracy
- large problems solved quickly in practice
- what "solve" means:
  - find **global** optimum within a given tolerance, or,
  - find **proof** (certificate) of infeasibility

Posynomial Functions

$$f(x_1, \ldots, x_n) = \sum_{k=1}^{t} c_k x_1^{\alpha_{1k}} x_2^{\alpha_{2k}} \cdots x_n^{\alpha_{nk}}$$

- f is a real-valued function of n real, positive variables $x = (x_1, x_2, \ldots, x_n)$.
- $c_j \geq 0$ and $\alpha_{ij} \in \Re$.
- If $t = 1$, f is called a ***monomial*** function.
- If f is a posynomial function, $\frac{1}{f}$ is called an ***inverse-posynomial*** function.
- Posynomials are *closed under sums, products, and nonnegative scaling.*

Posynomial Functions: Examples

- **Posynomial**

$$f_1 = 3x_1^{-0.3} + x_2^{1.3}x_3^{4.1}x_5 + .25x_1^{14}x_3^{-0.8}x_4^2$$

- **Mononomial**

$$f_2 = .25x_1^{14}x_3^{-0.8}x_4^2$$

- **Inverse Posynomial**

$$f_3 = \frac{1}{3x_1^{-0.3} + x_2^{1.3}x_3^{4.1}x_5 + .25x_1^{14}x_3^{-0.8}x_4^2}$$

Geometric Programming

$$\begin{array}{lll} \text{minimize} & f_0(x) & \\ \text{subject to} & f_i(x) \leq 1, & i = 1, \ldots, m \\ & g_i(x) = 1, & i = 1, \ldots, p \\ & x_i > 0, & i = 1, \ldots, n \end{array}$$

where f_i are *posynomial* functions and g_i are *monomial* functions.

The objective function $f_0(x)$ can be monomial or posynomial.

Geometric Programming

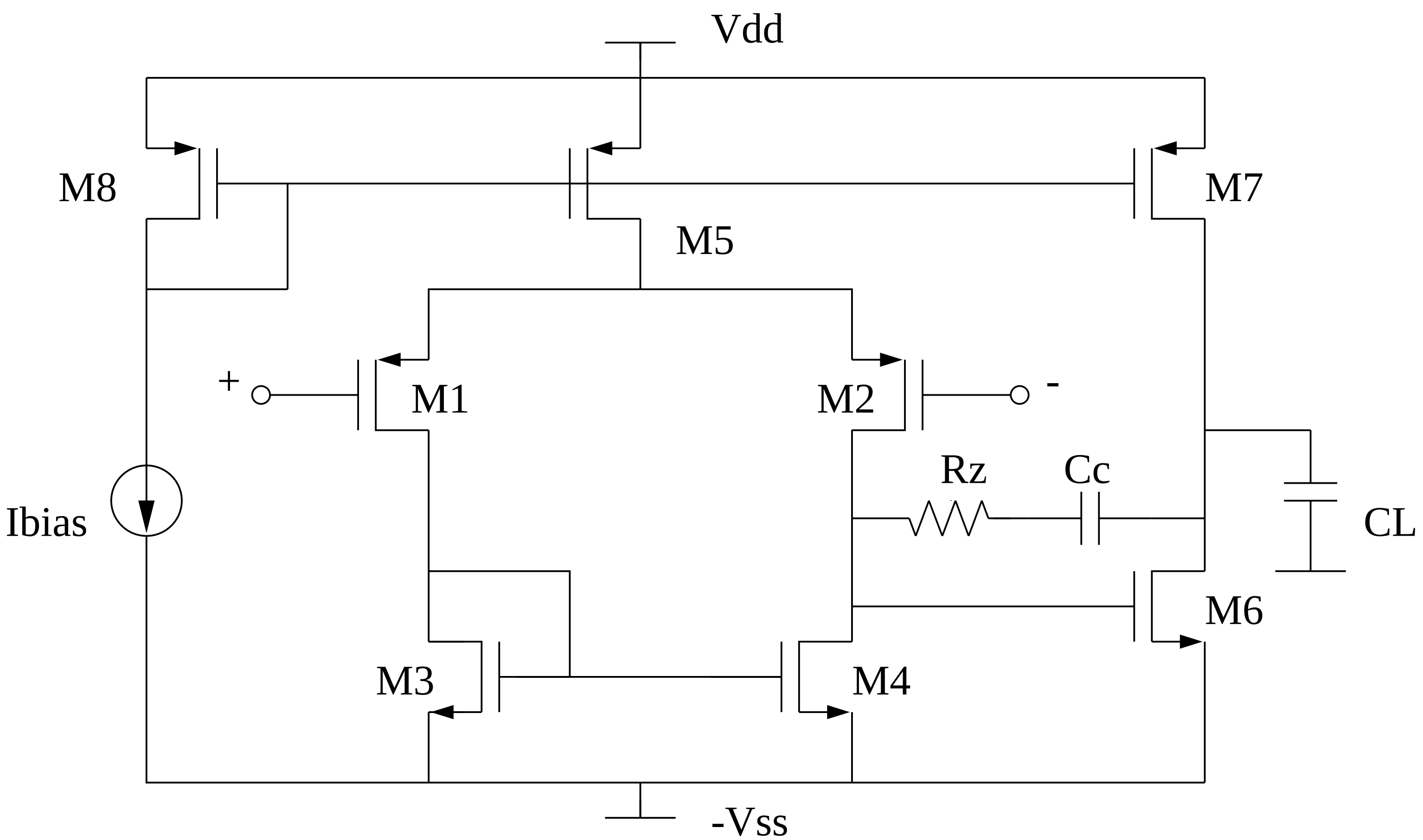
We can

- ***minimize*** any posynomial or monomial function
- or ***maximize*** any inverse-posynomial or monomial function

subject to

- ***upper-bounded*** posynomial functions
- ***lower-bounded*** inverse-posynomial functions
- ***upper and/or lower-bounded*** monomial functions.

Two-Stage Operational Amplifier



Dimension Constraints

- Minimum Device Sizes⟶**Monomial**

$$L_i \geq L_{\min} \quad \text{and} \quad \mathrm{W_i} \geq \mathrm{W_{min}}$$

- Area ⟶**Posynomial**

$$\text{Area} = \alpha_1 C_{\mathrm{c}} + \alpha_2 \sum_i W_i L_i.$$

- Systematic Input Offset Voltage ⟶**Monomial**

$$\frac{(W/L)_3}{(W/L)_6} = \frac{1}{2}\frac{(W/L)_5}{(W/L)_7}$$

$$\frac{(W/L)_4}{(W/L)_6} = \frac{1}{2}\frac{(W/L)_5}{(W/L)_7}.$$

Bias Conditions

- Additional variables $\longrightarrow$**Monomial**

$$I_5 = \frac{W_5 L_8}{L_5 W_8} I_{\mathrm{bias}} \qquad I_7 = \frac{W_7 L_8}{L_7 W_8} I_{\mathrm{bias}} \qquad I_1 = \frac{I_5}{2}$$

- Transistors in saturation $\longrightarrow$**Posynomial**
  (Transistors M_1 and M_2)

$$\sqrt{\frac{I_1 L_3}{\mu_{\mathrm{n}} C_{\mathrm{ox}}/2W_3}} \leq V_{\mathrm{cm,min}} + V_{\mathrm{ss}} - V_{\mathrm{TP}} - V_{\mathrm{TN}}$$

- Quiescent Power $\longrightarrow$**Posynomial**

$$P = (V_{\mathrm{dd}} + V_{\mathrm{ss}})(I_{\mathrm{bias}} + I_5 + I_7)$$

Transfer Function (I)

- Open Loop Gain $\longrightarrow$**Monomial**

$$A_v = \left(\frac{g_{\mathrm{m2}}}{g_{\mathrm{o2}} + g_{\mathrm{o4}}}\right)\left(\frac{g_{\mathrm{m6}}}{g_{\mathrm{o6}} + g_{\mathrm{o7}}}\right)$$

- 3-dB Cutoff Frequency $\longrightarrow$**Monomial**

$$\omega_{\mathrm{3dB}} = \frac{-g_{\mathrm{m1}}}{A_{\mathrm{v}} C_{\mathrm{c}}}$$

- Unity Gain Frequency $\longrightarrow$**Monomial**

$$\omega_{\mathrm{c}} = \frac{g_{\mathrm{m1}}}{C_{\mathrm{c}}}$$

Transfer Function (II)

- Phase Margin Conditions ⟶**Posynomial**

$$\frac{\omega_c}{p_2} + \frac{\omega_c}{z_1} \leq \frac{\pi}{2} - \mathrm{PM}_{\mathrm{min}}$$

or

$$\frac{\omega_c}{p_2} + \frac{\omega_c}{p_3} \leq \frac{\pi}{2} - \mathrm{PM}_{\mathrm{min}}$$

Slew Rate

- Slew Rate $\longrightarrow$**Posynomial**

$$\mathrm{SR} = \min\left(\frac{2\mathrm{I}_1}{\mathrm{C_c}}, \frac{\mathrm{I}_7}{\mathrm{C_c}+\mathrm{C_L}}\right)$$

A constraint on slew rate can be written as

$$\frac{C_{\mathrm{c}}}{2I_1} \leq \frac{1}{\mathrm{SR_{min}}}$$

$$\frac{C_{\mathrm{c}}+C_{\mathrm{L}}}{I_7} \leq \frac{1}{\mathrm{SR_{min}}}$$

Noise

- Input-referred Noise Spectral Density ⟶**Posynomial**

$$S_{\mathrm{i}}(f) = \frac{v_{in}^2}{\Delta f} \quad = \quad \frac{2K_p}{C_{\mathrm{ox}} W_1 L_1}\left(1 + \frac{K_n \mu_{\mathrm{n}} L_1^2}{K_p \mu_{\mathrm{p}} L_3^2}\right)\frac{1}{f}$$
$$+ \frac{16kT}{3\sqrt{2\mu_{\mathrm{p}} C_{\mathrm{ox}} (W/L)_1 I_1}}\left(1 + \sqrt{\frac{\mu_{\mathrm{n}}(W/L)_3}{\mu_{\mathrm{p}}(W/L)_1}}\right)$$

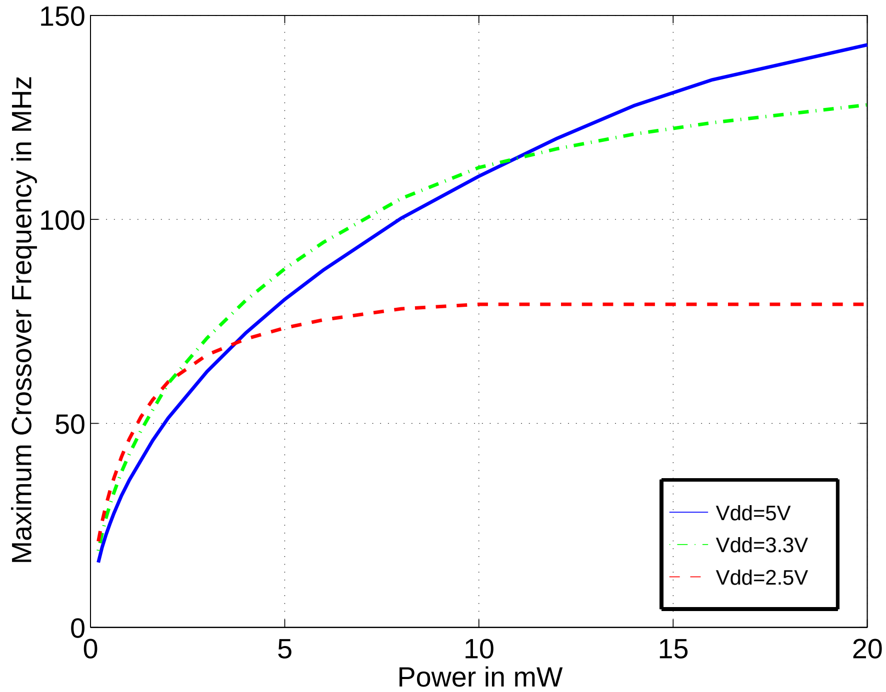
- Total Input-referred Noise⟶**Posynomial**

$$\overline{v_{\mathrm{INT}}^2} = \int_{f=0}^{f=f_{\mathrm{n}}} S_{\mathrm{i}}(f)\,df \approx \sum_{f=0}^{f=f_{\mathrm{n}}} S_{\mathrm{i}}(f)\Delta f$$

Other Constraints

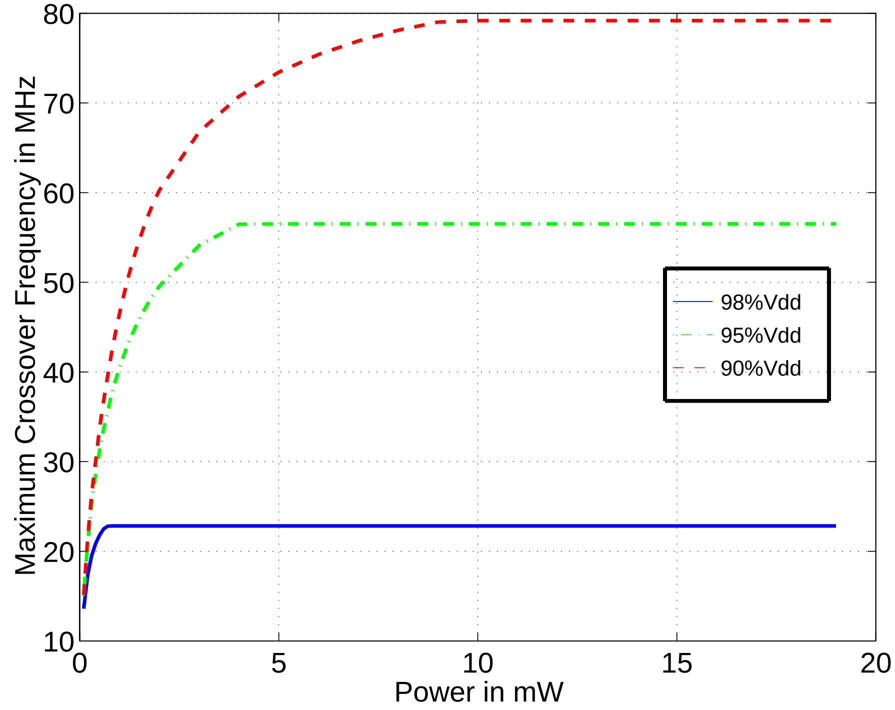
- Symmetry⟶**Monomial**
- Matching Conditions⟶**Monomial**
- CMRR ⟶**Monomial**
- Gate Overdrive ⟶**Monomial**
- Poles and Zeros ⟶**Inverse-Posynomial**
- . . .

Optimal Trade-off Curves (I)



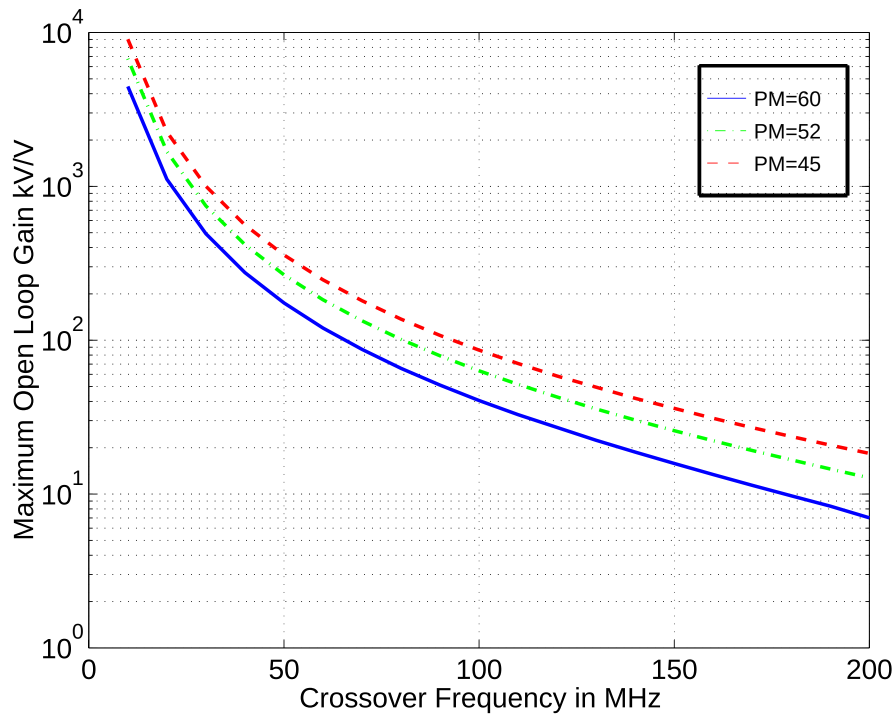
Unity gain bandwidth versus power for different supply voltages

Optimal Trade-off Curves (II)



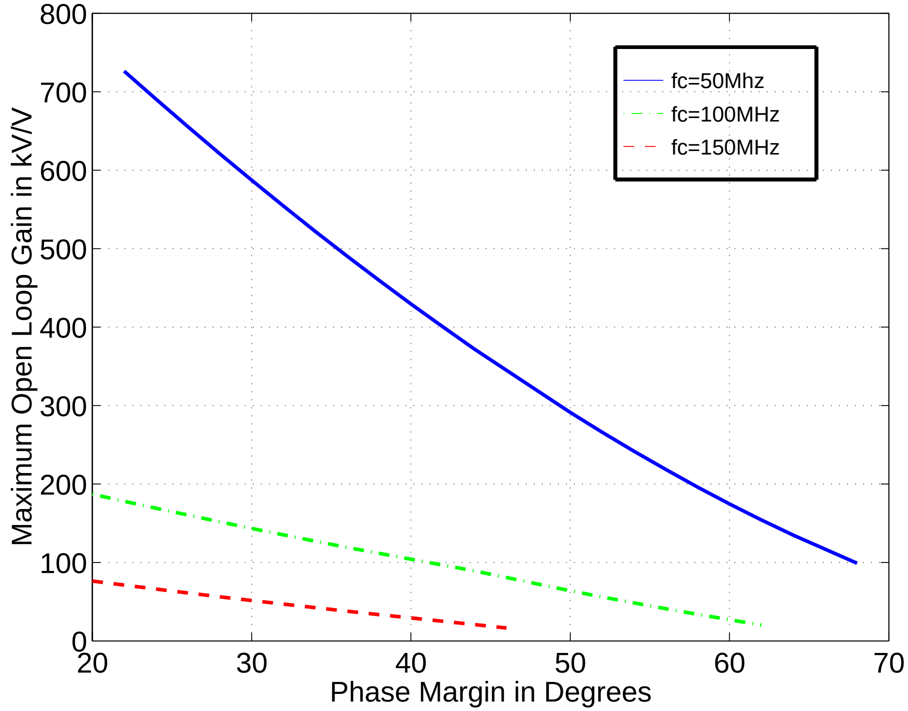
Maximum unity gain frequency versus power for different output voltage ranges

Optimal Trade-off Curves (III)



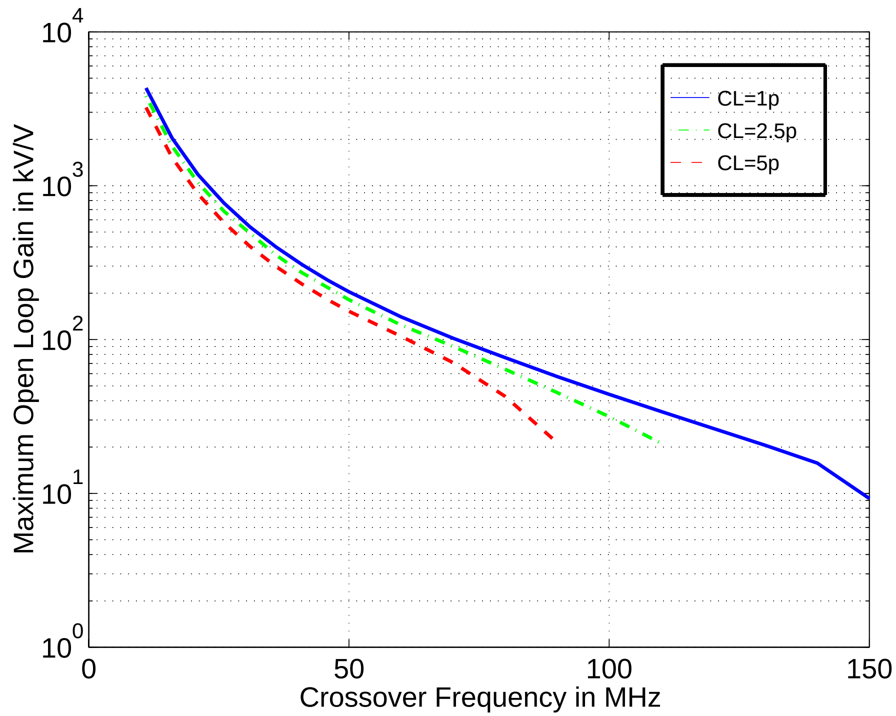
Maximum gain versus unity gain frequency for different phase margins

Optimal Trade-off Curves (IV)



Maximum open loop gain versus phase margin for different unity gain frequencies

Optimal Trade-off Curves (V)



Maximum open loop gain versus unity gain frequency for different load capacitances

Conclusions

Geometric Programming problems

- Arise in ***many*** important ***analog circuit*** design problems, in particular ***CMOS op-amp*** design.
- Can be ***(globally, efficiently) solved.***

We can

- ***automatically, directly from specifications, optimally design*** CMOS op-amps.

Acknowledgments

We gratefully acknowledge Edo Walks, who wrote the geometric programming code.

Final thought . . .

. . . the great watershed in optimization isn't between linearity and nonlinearity, but convexity and nonconvexity.
— R. Rockafellar, SIAM Review 1993