CMOS Injection-locked
Ring Oscillator
Frequency Dividers

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Outline

• **Goals**
  • Ring Oscillator Overview
  • Injection Locking Theory
  • Circuit Implementation
  • Measured Results
  • Conclusion
CMOS Injection-locked Ring Oscillator Frequency Dividers

Goals

- Appreciate the trade-offs of low-power frequency synthesis
- Understand the operation of Ring Oscillators
- Understand the Injection-locking mechanism
- Grasp the limitations of Injection-locked Frequency Dividers
- Design Injection-locked Frequency Divider using a Ring
CMOS Injection-locked Ring Oscillator Frequency Dividers

Motivation

A short-haul, low-power, radio-on-a-chip (RoC) that requires no external components can enable novel applications that are not economically feasible otherwise.

APPLICATIONS

- Ambulatory health monitoring and biotelemetry
- Building and environmental monitoring
- Distribution and retail inventory management
- Wireless Internet access
- Home and factory automation

ISSUES

- A significant portion of the power budget is allocated to the generation of the RF local oscillator.
- Requires a low-power, completely integrated frequency synthesizer.
Low-power Frequency Synthesis

- Frequency synthesizers are implemented using Phase-locked Loops (PLLs).

- Major sources of power dissipation are the Voltage-controlled Oscillator (VCO) and the Frequency Divider.
The VCO’s power dissipation is determined by the frequency of operation and the phase noise performance required.

A PLL tracks phase noise of the reference within its loop bandwidth, relaxing the close-in phase noise requirements of the VCO.
POWER INCREASES WITH DIVISION RATIO

- Better understanding of low-power techniques for frequency division is essential to reduce the overall power dissipation of integrated frequency synthesizers.
- We propose a technique in which power decreases with division ratio. Are you interested?
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How can a circuit oscillate?

• BADLY-DESIGNED FEEDBACK AMPLIFIER

![Diagram of a feedback amplifier]

Closed-loop Gain

\[ A_{vl}(j\omega) = \frac{H(j\omega)}{1 + H(j\omega)} \]

IF \( H(j\omega_o) = -1 \) THEN \( \lim_{\omega \to \omega_o} A_{vl}(j\omega) = \infty \)

BARKHAUSEN CRITERIA

• Necessary conditions for oscillation
• Total phase shift around the loop is 360\(^\circ\)
• Amplifies its own noise at \( \omega_0 \)
• Positive feedback or “regeneration”

\[
|H(j\omega_o)| \geq 1 \\
\angle H(j\omega_o) = 180^\circ
\]
What is a Ring Oscillator?

- A RING OSCILLATOR CONSISTS OF A NUMBER OF GAIN STAGES IN A FEEDBACK LOOP

\[ H_S(j\omega) = \frac{H_O}{1 + j\omega/\omega_P} \]

\[ H_O = -g_m R_L \]

\[ \omega_P = \frac{1}{R_L C_L} \]

\[ C_L = C_{GS} + (2 + g_m R_L) C_{GD} + C_{DB} \]

- Neglect feedforward zero \(+g_m/C_{GD}\)
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Evolution of the Ring Oscillator

1 STAGE

$$H(j\omega) = \frac{H_O}{1 + j\omega/\omega_P}$$

$$\angle H(j\omega)|_\infty = 90^\circ$$

INSUFFICIENT PHASE

2 STAGES

$$H(j\omega) = \frac{H_O^2}{(1 + j\omega/\omega_P)^2}$$

$$\angle H(j\omega)|_\infty = 180^\circ$$

INSUFFICIENT GAIN

3 STAGES

$$\angle H(j\omega)|_\infty = 270^\circ$$

IT WORKS!
4-STAGE DIFFERENTIAL RING OSCILLATOR

- EACH STAGE CONTRIBUTES 45° @ $\omega_O$
- EACH POLE CONTRIBUTES 45° @ $\omega_P$

IN GENERAL

$$H(j\omega) = \frac{H_O^n}{(1 + j\omega/\omega_P)^n}$$

PHASE CONDITION

$$\angle H(j\omega_o) = n \cdot \tan\left(\frac{\omega_o}{\omega_P}\right) = \pi$$

GAIN CONDITION

$$H_O \geq \sqrt{1 + \tan\left(\frac{\pi}{n}\right)^2}$$

LARGE SWING (AMPLITUDE LIMITED)

$$f_{osc} = \frac{1}{2nT_D} < f_o \quad \text{WHERE} \quad T_D \propto \tau_P$$
Voltage-Controlled Ring Oscillator

- Delay of each stage $T_D$ is tuned by control input.
- Change delay by varying capacitance or resistance at each stage.

$$T_D \propto R_L C_L \quad \omega_p = \frac{1}{R_L C_L}$$

**CAPACITIVE TUNING**

**TRIODE LOAD**

**DIODE-CONNECTED LOAD**

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SYMMETRIC LOAD [MANEATIS’94]

- Use buffers with replica-feedback biasing.
- $V_C$ changes the bias $I_C$ of the buffers.
- Replica bias ensures load symmetry by forcing the maximum single-ended swing $V_S = V_{dd} - V_C$
- Good supply noise rejection
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What is Injection Locking? [Adler 1946]

OSCILLATOR OUTPUT SYNCHRONIZES TO INJECTED SIGNAL

\[ V_{I@\omega_{I}} \quad V_{E} \quad H(j\omega) \quad V_{O@\omega} \]

\[ |H(j\omega_{O})| \geq 1 \]
\[ \angle H(j\omega_{O}) = 180^\circ \]

ASSUME \( \omega_{I} \approx \omega_{O} \)
\( V_{I} \ll V_{O} \)
Fast Amplitude Limiting Mechanism

- **IF** \( V_{I} = 0 \) **THEN** \( \omega = \omega_{O} \)
- **IF** \( V_{I} \neq 0 \) **THEN** \( \omega \) shifts from free-running frequency, \( \omega_{O} \).
- Frequency shift is proportional to \( V_{I}/V_{O} \).
Locking Range of Injection-locked Oscillator

\[
\left| \frac{\Delta \omega_O}{\omega_O} \right| < \frac{V_I}{V_O} \cdot \frac{2}{n \sin \left( \frac{2\pi}{n} \right)}
\]

\[
\Delta \omega_O = \omega_O - \omega_I
\]

- Proportional to \( V_I / V_O \).
- Inversely proportional to number of stages, \( n \).
- Only valid for \( V_I \ll V_O \).
Superharmonic Injection Locking

• An oscillator can be injection-locked to a harmonic of the free-running oscillation frequency.

• This principle is used by all regenerative frequency dividers.

• Regenerative dividers are commonly used in applications where the frequency of operation is very high, beyond what can be achieved with flip-flop based circuits.

• Efforts at frequencies beyond 5 GHz have been reported using injection-locking to implement divide-by-2 prescalers in CMOS, and Si-BJT technologies.

• Commonly used at mm-wave frequencies in GaAs and SiGe

We want to exploit injection locking to achieve low-power frequency division.
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Regenerative Divider [Miller 1939]

- Can achieve division ratios greater than two by using a frequency multiplier in the feedback.
- Frequency multiplier can represent non-linearities present in the circuit.
- We can describe an Injection-locked Frequency Divider (ILFD) using a generalized mixer-

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Generalized Model for Injection-locked Divider

**MOS Injection-locked Ring Oscillator Frequency Dividers**

**Mixer**
- Single-balanced mixer based on a differential-pair
- Injected $\omega_{RF}$ into the tail device ("Injector"), which produces an RF current that adds to $I_{BIAS}$
- RF current may include a DC component and all harmonics of $\omega_{RF}$ For now, we will ignore this effect.

**Loop Filter**
- Mixer products are low-pass filtered and amplified by $H(j\omega)$
- Suppress mixer products $> \omega_0$
- For small $n$, the output voltage $V_O$ is sinusoidal.

$I_{TAIL} = I_{RF} \cos(\omega_{RF}t + \alpha) + I_{BIAS}$

$DC + \omega_{RF}$

**RF Port**

$\omega_0, 3\omega_0, 5\omega_0 \ldots$

**LO Port**

Differential Pair's Non-linearity

$n$-stage LPF

$H(j\omega)$

$\omega_0 = \omega_{RF}/M$

$V_O \cos(\omega_0 t)$
Use describing function analysis to determine the open-loop transfer characteristic’s phase and magnitude components.

ASSUMPTIONS

- If $V_O$ is large, then the injection locking dynamics are determined by the phase relationship around the loop (phase-limited) and therefore we can ignore the amplitude expression.

- A large amplitude is also required to excite the Mixer’s LO port non-linearity, which is the mechanism that makes possible division ratios greater than two.
Mixer

\[ DC + \omega_{RF} \]

\[ \omega_0, 3\omega_0, 5\omega_0 \ldots \]

\[ I_{TAIL} \]

\[ \Pi(t) \cdot [I_{RF} \cdot \cos(\omega t + \alpha) + I_{BIAS}] \]

\[ \Delta I \]

\[ 2I_{RF} \]

\[ -V_{SAT} \]

\[ V_{SAT} \]

\[ \Delta V \]

\[ I_{BIAS} \]

\[ -I_{BIAS} \]

ASSUME \( V_O >> V_{SAT} \) (SWITCH HARD)

Fourier Coefficients of Mixing Function (Square Wave)

\[ C_k = \begin{cases} 
\frac{1}{k\pi} \cdot (-1)^{(k-1)/2} & \text{for } k = \text{odd} \\
0 & \text{otherwise} 
\end{cases} \]

- The differential-pair’s transfer characteristic is non-linear with odd symmetry.
- When excited by \( \omega_0 \), the mixer’s non-linearity produce odd harmonics at \( 3\omega_0, 5\omega_0 \), etc.
- The total current \( I_{TAIL} \) is modulated by \( \omega_0 \)
A CMOS injection-locked ring oscillator frequency divider is discussed. The loop filter is a crucial component in this system. The transfer function $H(j\omega)$ is given by:

$$H(j\omega) = \frac{H_O^n}{\left(1 + j\frac{\omega}{\omega_o}\tan\left(\frac{\pi}{n}\right)\right)^n}$$

where $\omega_0$ is the frequency of the free-running oscillator. Each stage contributes $\pi/n$ to the phase.
WRITE PHASE EXPRESSION AROUND THE LOOP

\[
\tan\left(\frac{\eta_i \left(C_{M-1} - C_{M+1}\right) \sin \alpha}{C_1 + \eta_i \left(C_{M-1} + C_{M+1}\right) \cos \alpha}\right) = \angle H(j \omega) - \pi
\]

\[
\eta_i = \frac{I_{RF}}{2I_{BIAS}}
\]

LOCKING RANGE

\[
\Delta \omega \equiv \frac{4}{n \sin\left(\frac{2\pi}{n}\right)} \tan\left(\frac{k_0}{\sqrt{1 - k_1^2}}\right)
\]

\[
k_0 = \eta_i \left|\frac{C_{M-1} - C_{M+1}}{C_1}\right|
\]

\[
k_1 = \eta_i \left|\frac{C_{M-1} + C_{M+1}}{C_1}\right|
\]

Trade-offs

- The locking range is a function of injection efficiency \(\eta_i\), and the magnitude of the Fourier coefficients \(C_{M-1}\) and \(C_{M+1}\).
- For small values of injected signal the locking range increases linearly with the injected signal strength.
1. **Limited Mixer Gain** - Switching function is not a square wave. (Swing ratio should be large, \( \rho_s = V_0/V_{SAT} \geq 1 \))

   - As \( \rho_s \) gets smaller, the square wave assumption is no longer valid and the coefficient ratios \( C_k/C_1 \) are significantly smaller.

2. **Limited Injection Efficiency** - Due to short-channel effects, velocity saturation, device non-linearity.

   - Due to Injector non-linearities, \( I_{DC} \) rises for large injected signals (\( I_{DC} > I_{BIAS} \)). An increase of \( I_{DC} \) also affects \( V_{SAT} \), reducing the swing ratio.

\[
\eta_i = \frac{I_{RF}}{2I_{DC}} = \frac{V_{RF}}{2V_{OD, TAIL}} \cdot \gamma \quad I_{DS} = K \cdot (V_{RF} + V_{OD, TAIL})^\gamma
\]

3. **Mixer Tail Node Parasitics** - Due to tail drain junction and diffpair source junction.

   - Parasitic capacitance at the drain of the tail device provides a shunt path for \( I_{RF} \) reducing \( \eta_i \) at high frequencies.
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5-stage Ring Oscillator, modulo-8 Divider

(a) Ideal (phase-limited) case
(b) Compression due to Injector non-linearity
(c) Effect of Injector non-linearity and drain junction parasitics (50% RF current loss)
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5-stage Injection-locked Ring Oscillator Divider

- Modified cross-coupled symmetric load buffers were used for their good supply noise rejection and low 1/f noise upconversion characteristics.
- We injected the RF signal at the tail current source of the first buffer, using it as a single-balanced mixer.
- The buffer stages behave as the multipole filter \(H(j\omega)\) that contribute the gain and phase shift required to sustain the oscillation.
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Two ring oscillators were designed, with 3 and 5 buffer stages respectively.

Layout is symmetrical and load balanced to avoid any skewing between the phases.

- 0.24-\(\mu\text{m}\) CMOS
- 0.012 mm\(^2\) of area
Results

### Measurements

<table>
<thead>
<tr>
<th></th>
<th>5-stage ILFD</th>
<th>3-stage ILFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injected Frequency</td>
<td>1.0 GHz</td>
<td>2.8 GHz</td>
</tr>
<tr>
<td>Free-running Frequency</td>
<td>125 MHz</td>
<td>700 MHz</td>
</tr>
<tr>
<td>Phase Noise@100KHz</td>
<td>-110 dBC/Hz</td>
<td>-106 dBC/Hz</td>
</tr>
</tbody>
</table>

**Locking Range**

- Modulo-2: 12.7 MHz (-3dBm) 125 MHz (-3dBm)
- Modulo-4: 32 MHz (-3dBm) 56 MHz (-5dBm)
- Modulo-6: 17 MHz (-3dBm) no-lock
- Modulo-8: 20 MHz (-3dBm) no-lock

**Power dissipation**

<table>
<thead>
<tr>
<th></th>
<th>5-stage ILFD</th>
<th>3-stage ILFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vdd</td>
<td>1.5 V</td>
<td>3.0 V</td>
</tr>
<tr>
<td>Icore</td>
<td>233 µA</td>
<td>331 µA</td>
</tr>
<tr>
<td>Ibias</td>
<td>108 µA</td>
<td>661 µA</td>
</tr>
<tr>
<td>Core power</td>
<td>350 µW</td>
<td>993 µW</td>
</tr>
<tr>
<td>Power efficiency</td>
<td>2.86 GHz/mW</td>
<td>2.82 GHz/mW</td>
</tr>
</tbody>
</table>

- Swing is smaller than expected, hence the locking range is smaller than predicted.
- Locking range is not symmetric around the free-running frequency of the ILFD, specially at higher injected power levels. This behavior is due to the increase of \( I_{DC} \) with the injected signal.
Power Efficiency of Injection-locked Ring Oscillator

- Power efficiency is the ratio of the divider’s maximum operation frequency to its power dissipation expressed in GHz/mW.
- To achieve a fair comparison of the available data, only the “core” divider circuit is taken into consideration.
- 5-stage (mod-8), 2.86 GHz/mW @1GHz.
- 3-stage, (mod-4), 2.82 GHz/mW @2.8GHz.

**EXCEED ALL PUBLISHED RESULTS AT COMPARABLE FREQUENCIES**
What We Learned

- Need to scale down the Injector to lower the parasitics, thus increasing the injection efficiency.
- The tail node parasitics can also be cancelled by resonating with an inductor (shunt-peaking), but this is not practical at sub-GHz frequencies.
- Increase the output swing and the $W/L$ ratio of the Injector, hence increasing the swing ratio. This should be weighted against the resultant increase in parasitic capacitance and power dissipation.
- While a flip-flop based divider uses more power as we add more stages, the injection-locked divider uses less power for higher division ratios (every stage is operating at $\omega_0$).
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Conclusion

- Reviewed the operation of voltage-controlled CMOS ring oscillators.

- Described the injection locking mechanism and how it applies to CMOS ring oscillators.

- Showed the design of frequency dividers that can operate up to 2.8-GHz by exploiting the injection locking phenomena in differential CMOS ring oscillators.

- Showed measured results for 1-GHz and 2.8-GHz injection-locked frequency dividers fabricated in a 0.24-μm CMOS technology.

- Achieved the highest power efficiency (2.86 GHz/mW) ever
References


What is Phase Noise?

- Undesirable phase fluctuations due to intrinsic device noise
- Output power is not concentrated at the carrier frequency alone

- Phase noise is represented as a ratio of power in 1Hz bandwidth in one sideband to the power of the carrier.
- Specified in dBc/Hz at a frequency offset from the carrier.
**ASSUME** \( \omega_I \approx \omega_O \) AND \( V_I \ll V_O \)

\[
\phi = \angle H(j\omega) + \pi
\]

**LINEARIZE PHASE**

\[
\phi = \frac{d\phi}{d\omega} (\omega - \omega_O)
\]

\[\angle H(j\omega) \]

\[\omega_o \quad \omega \]

\[\pi \quad d\phi/d\omega \]

\[
\omega = \omega_I + \frac{d\alpha}{dt} \quad \phi = -\frac{V_I}{V_O} \sin \alpha
\]

\[
\omega = -\frac{V_I}{V_O} \sin \alpha + \omega_O \frac{d\phi}{d\omega}
\]

- Output frequency shifts from free-running frequency
- **IF** \( V_I = 0 \) **THEN** \( \phi = 0 \) and \( \omega = \omega_O \)
Locking Range of Injection-locked Oscillator

1. FREQUENCY SHIFT

\[ \omega = \frac{-V_I/V_O \sin \alpha}{d\phi/d\omega} + \omega_O \]
\[ \omega = \omega_I + \frac{d\alpha}{dt} \]

2. DIFFERENTIAL EQUATION

\[ \frac{d\alpha}{dt} = \frac{-V_I/V_O \sin \alpha}{d\phi/d\omega} + \Delta \omega_O \]
\[ \Delta \omega_O = \omega_O - \omega_I \]

3. LINEARIZE PHASE OF \( H(j\omega) \)

\[ \frac{d\phi}{d\omega} \approx \frac{n}{2\omega_O} \sin \left( \frac{2\pi}{n} \right) \]

4. STEADY-STATE SOLUTION

FOR \( \frac{d\alpha}{dt} = 0 \)

\[ \left| \frac{\Delta \omega_O}{\omega_O} \right| < \frac{V_I}{V_O} \cdot \frac{2}{n \sin \left( \frac{2\pi}{n} \right)} \]

LOCKING RANGE