CMOS Operational Amplifier Design and Optimization via Geometric Programming

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Main Idea

- Many op-amp design problems can be cast as a special type of convex optimization problems.
- Such problems can appear very difficult, but can be solved very efficiently by recently developed interior-point (IP) methods.
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Outline

- Convex Optimization
- Posynomial Functions
- Geometric Programming
- Application to a Two-Stage Operational Amplifier
- Results
- Conclusions
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Convex Optimization

minimize $f_0(x)$

subject to $f_1(x) \leq 0, \ldots, f_L(x) \leq 0$, $Ax = b$

- $x \in \mathbb{R}^n$ is optimization variable
- $f_i$ are convex:

  for $0 \leq \lambda \leq 1$,
  
  $f_i(\lambda x + (1-\lambda)y) \leq \lambda f_i(x) + (1-\lambda)f_i(y)$
Convex Optimization problems are fundamentally tractable

- computation time is small, grows gracefully with problem size and required accuracy
- large problems solved quickly in practice
- what “solve” means:
  - find global optimum within a given tolerance, or,
  - find proof (certificate) of infeasibility
Posynomial Functions

\[ f(x_1, \ldots, x_n) = \sum_{k=1}^{t} c_k x_1^{\alpha_{1k}} x_2^{\alpha_{2k}} \cdots x_n^{\alpha_{nk}} \]

- \( f \) is a real-valued function of \( n \) real, positive variables \( x = (x_1, x_2, \ldots, x_n) \).
- \( c_j \geq 0 \) and \( \alpha_{ij} \in \mathbb{R} \).
- If \( t = 1 \), \( f \) is called a \textit{monomial} function.
- If \( f \) is a posynomial function, \( \frac{1}{f} \) is called an \textit{inverse-posynomial} function.
- Posynomials are \textit{closed under sums, products, and nonnegative scaling}. 
Posynomial Functions: Examples

- **Posynomial**

  \[ f_1 = 3x_1^{-0.3} + x_2^{1.3} x_3^{4.1} x_5 + 0.25x_1^{14} x_3^{-0.8} x_4^{2} \]

- **Mononomial**

  \[ f_2 = 0.25x_1^{14} x_3^{-0.8} x_4^{2} \]

- **Inverse Posynomial**

  \[ f_3 = \frac{1}{3x_1^{-0.3} + x_2^{1.3} x_3^{4.1} x_5 + 0.25x_1^{14} x_3^{-0.8} x_4^{2}} \]
Geometric Programming

\[
\text{minimize} \quad f_0(x) \\
\text{subject to} \quad f_i(x) \leq 1, \quad i = 1, \ldots, m \\
g_i(x) = 1, \quad i = 1, \ldots, p \\
x_i > 0, \quad i = 1, \ldots, n
\]

where \( f_i \) are \textit{posynomial} functions and \( g_i \) are \textit{monomial} functions.

The objective function \( f_0(x) \) can be monomial or posynomial.
We can

- *minimize* any posynomial or monomial function
- or *maximize* any inverse-posynomial or monomial function

subject to

- *upper-bounded* posynomial functions
- *lower-bounded* inverse-posynomial functions
- *upper and/or lower-bounded* monomial functions.
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Two-Stage Operational Amplifier

\[ V_{dd} \]

\[ M8 \]

\[ M5 \]

\[ M7 \]

\[ + \]

\[ M1 \]

\[ M2 \]

\[ Rz \]

\[ Cc \]

\[ M3 \]

\[ M4 \]

\[ M6 \]

\[ -V_{ss} \]

\[ CL \]

\[ -V_{ss} \]

\[ M1 \]

\[ Ibias \]

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Dimension Constraints

- **Minimum Device Sizes** → Monomial
  \[ L_i \geq L_{\text{min}} \quad \text{and} \quad W_i \geq W_{\text{min}} \]

- **Area** → Posynomial
  \[ \text{Area} = \alpha_1 C_c + \alpha_2 \sum W_i L_i. \]

- **Systematic Input Offset Voltage** → Monomial
  \[ \frac{(W/L)_3}{(W/L)_6} = \frac{1}{2} \frac{(W/L)_5}{(W/L)_7}, \]
  \[ \frac{(W/L)_4}{(W/L)_6} = \frac{1}{2} \frac{(W/L)_5}{(W/L)_7}. \]
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Bias Conditions

- Additional variables $\rightarrow$ Monomial

\[
I_5 = \frac{W_5L_8}{L_5W_8} I_{bias} \quad I_7 = \frac{W_7L_8}{L_7W_8} I_{bias} \quad I_1 = \frac{I_5}{2}
\]

- Transistors in saturation $\rightarrow$ Posynomial
  (Transistors \( M_1 \) and \( M_2 \))

\[
\sqrt{\frac{I_1L_3}{\mu_n C_{ox}/2W_3}} \leq V_{cm,\text{min}} + V_{ss} - V_{TP} - V_{TN}
\]

- Quiescent Power $\rightarrow$ Posynomial

\[
P = (V_{dd} + V_{ss})(I_{bias} + I_5 + I_7)
\]
Transfer Function (I)

- Open Loop Gain $\rightarrow$ Monomial

$$A_v = \left(\frac{g_{m2}}{g_{o2} + g_{o4}}\right) \left(\frac{g_{m6}}{g_{o6} + g_{o7}}\right)$$

- 3-dB Cutoff Frequency $\rightarrow$ Monomial

$$\omega_{3dB} = \frac{-g_{m1}}{A_v C_c}$$

- Unity Gain Frequency $\rightarrow$ Monomial

$$\omega_c = \frac{g_{m1}}{C_c}$$
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Transfer Function (II)

- Phase Margin Conditions → Posynomial

\[
\frac{\omega_c}{p_2} + \frac{\omega_c}{z_1} \leq \frac{\pi}{2} - \text{PM}_{\text{min}}
\]

or

\[
\frac{\omega_c}{p_2} + \frac{\omega_c}{p_3} \leq \frac{\pi}{2} - \text{PM}_{\text{min}}
\]
Slew Rate

- Slew Rate $\rightarrow$ Posynomial

\[
SR = \min \left( \frac{2I_1}{C_c}, \frac{I_7}{C_c + C_L} \right)
\]

A constraint on slew rate can be written as

\[
\frac{C_c}{2I_1} \leq \frac{1}{SR_{\text{min}}}
\]

\[
\frac{C_c + C_L}{I_7} \leq \frac{1}{SR_{\text{min}}}
\]
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Noise

- Input-referred Noise Spectral Density → Posynomial

\[
S_i(f) = \frac{v_{in}^2}{\Delta f} = \frac{2K_p}{C_{ox}W_1L_1} \left( 1 + \frac{K_n \mu_n L_1^2}{K_p \mu_p L_3^2} \right) \frac{1}{f} \\
+ \frac{16kT}{3\sqrt{2\mu_p C_{ox}(W/L)_1 I_1}} \left( 1 + \sqrt{\frac{\mu_n(W/L)_3}{\mu_p(W/L)_1}} \right)
\]

- Total Input-referred Noise → Posynomial

\[
\overline{v_{INT}^2} = \int_{f=0}^{f=f_n} S_i(f) \, df \approx \sum_{f=0}^{f=f_n} S_i(f) \Delta f
\]
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Other Constraints

- Symmetry $\rightarrow$ Monomial
- Matching Conditions $\rightarrow$ Monomial
- CMRR $\rightarrow$ Monomial
- Gate Overdrive $\rightarrow$ Monomial
- Poles and Zeros $\rightarrow$ Inverse-Posynomial
- ...
Optimal Trade-off Curves (I)

Unity gain bandwidth versus power for different supply voltages

Vdd = 5V
Vdd = 3.3V
Vdd = 2.5V
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Optimal Trade-off Curves (II)

Maximum unity gain frequency versus power for different output voltage ranges
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Optimal Trade-off Curves (III)

Maximum gain versus unity gain frequency for different phase margins

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**Optimal Trade-off Curves (IV)**

Maximum open loop gain versus phase margin for different unity gain frequencies
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Optimal Trade-off Curves (V)

Maximum open loop gain versus unity gain frequency for different load capacitances
Conclusions

*Geometric Programming* problems

- Arise in *many* important *analog circuit* design problems, in particular *CMOS op-amp* design.

- Can be *(globally, efficiently) solved.*

We can

- *automatically, directly from specifications, optimally design* CMOS op-amps.
We gratefully acknowledge Edo Walks, who wrote the geometric programming code.
Final thought . . .

... the great watershed in optimization isn’t between linearity and nonlinearity, but convexity and nonconvexity.
— R. Rockafellar, SIAM Review 1993